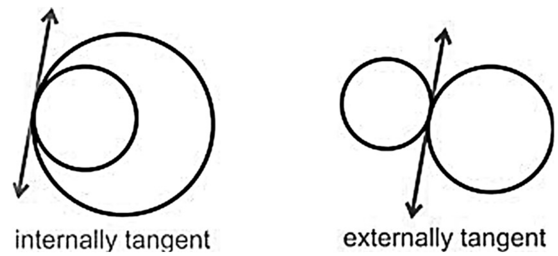
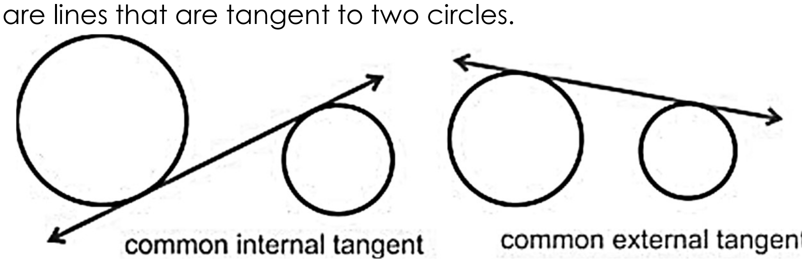
Tangent and Chord Properties

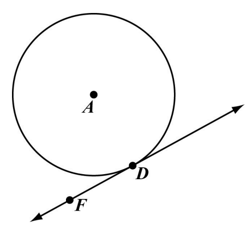
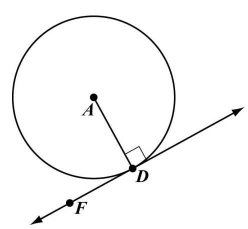
On Day 1, you learned that tangent lines intersect a circle in exactly one place. This leads to several theorems about tangent lines.

Tangent Circles are two coplanar circles that intersect at exactly one point. They may intersect internally or externally.

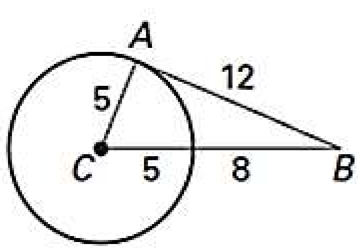


Common Tangent Lines are lines that are tangent to two circles.

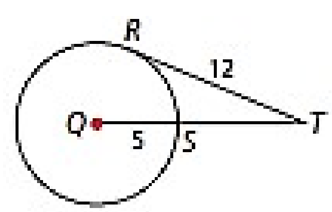


Name	Theorem	Hypothesis	Conclusion
Perpendicular Tangent Theorem	If a line is tangent to a circle, then it is **perpendicular** to the radius drawn to the point of tangency.		
Converse of Perpendicular Tangent Theorem	If a line is **perpendicular** to a radius of a circle at a point on the circle, then the line is tangent to the circle.		

Example: Is AB tangent to Circle C?

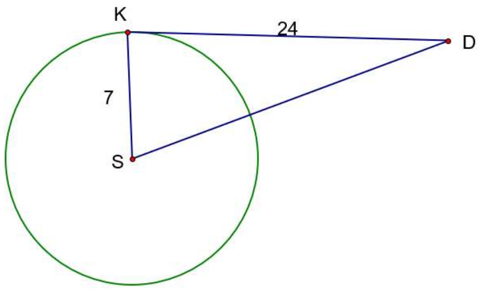


Example: Find ST.

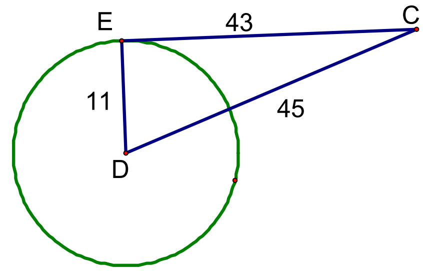


Try:

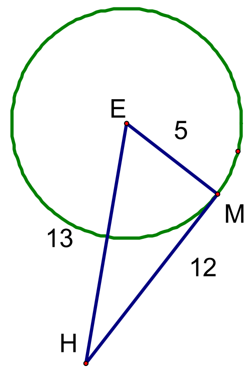
1. If $\overline{KD}$ is tangent to circle S, find the length of $\overline{SD}$.

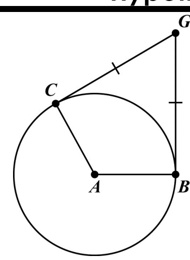


2. Determine if $\overline{EC}$ is tangent to circle D. Explain your answer.

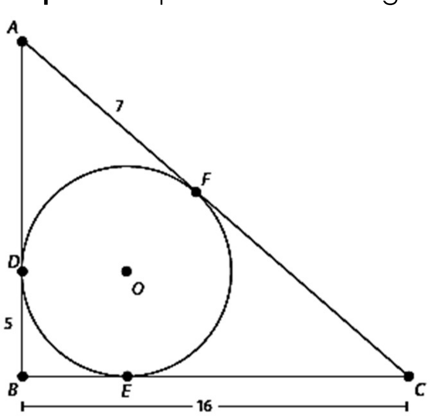


3. Is segment MH tangent to circle E? Justify your answer.

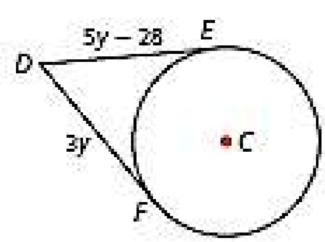


Name	Theorem	Hypothesis	Conclusion
Tangent Segments Theorem	If two segments are tangent to a circle from the same external point, then the segments are congruent.		

Example: Find perimeter of triangle ABC.

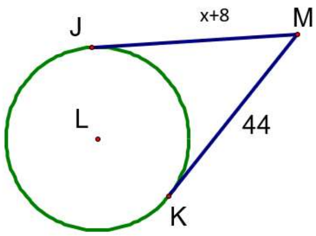


Example: Find DF if you know that DF and DE are tangent to $\odot C$.

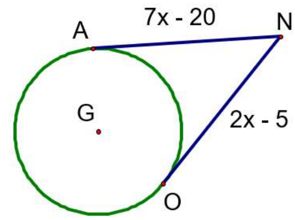


Try:

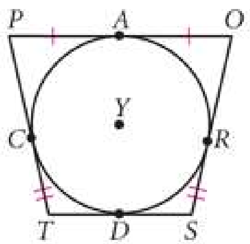
1. $\overline{JM}$ and $\overline{MK}$ are tangent to circle L.
   Find the value of x.



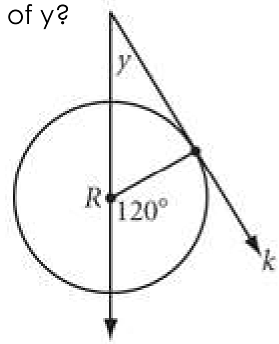
2, $\overline{NA}$ and $\overline{NO}$ are tangent to circle G.
   Find the value of x.



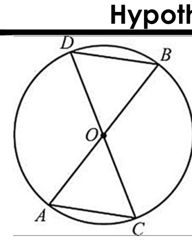
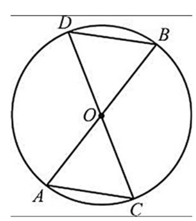
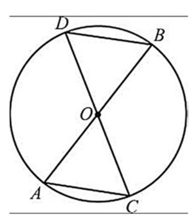
3. Quadrilateral POST is circumscribed about circle Y.
   OR = 13 in. and ST = 12 in. Find the perimeter of POST.



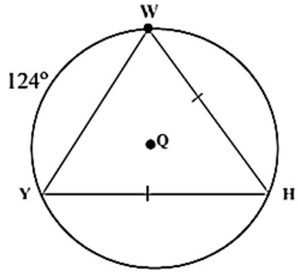
4. Ray k is tangent to circle R.
   What is the value of y?



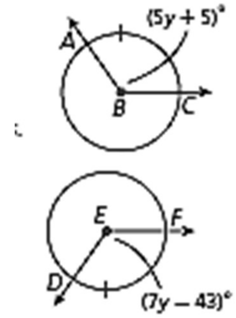
Chord Properties

Name	Theorem	Hypothesis	Conclusion
Congruent Angle-Congruent Chord Theorem	Congruent central angles have congruent chords.		
Congruent Chord-Congruent Arc Theorem	Congruent chords have congruent arcs.		
Congruent Arc-Congruent Angle Theorem	Congruent arcs have congruent central angles.		

Example: Find the measure of arc HY and HYW.

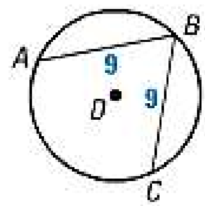


Example: Find the measure of angle DEF.



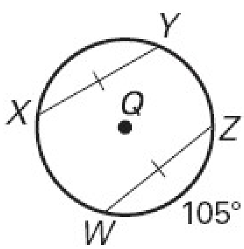
Example: Use the diagram of $\odot D$.

1. If $m\widehat{AB} = 110°$, find $m\widehat{BC}$.
2. If $m\widehat{AC} = 150°$, find $m\widehat{AB}$.

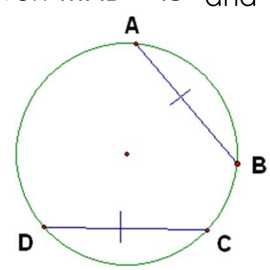


Try:

1. Find the measure of arc YZ if the measure of arc XW = 95°

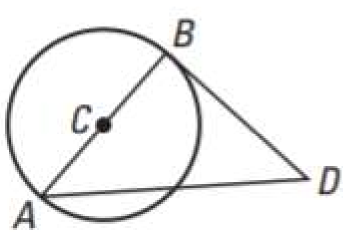


2. Given $m\widehat{AB} = 45^\circ$ and $m\widehat{BC} = 22^\circ$.



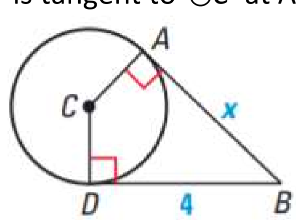
Skills Practice

1. In the diagram below, AB = BD = 5 and AD = 7. Is $\overline{BD}$ tangent to $\odot C$? Explain.

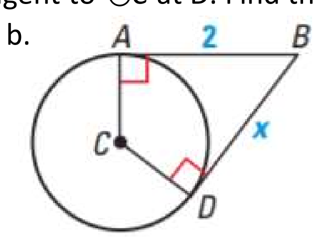


2. $\overline{AB}$ is tangent to $\odot C$ at A and $\overline{DB}$ is tangent to $\odot C$ at D. Find the value of x.

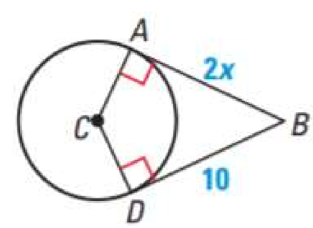
a.



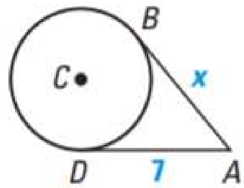
b.

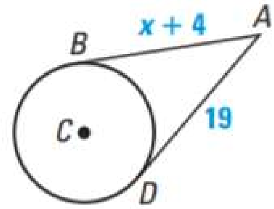


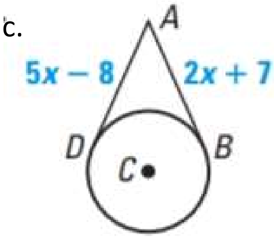
c.



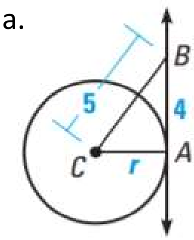
3. $\overline{AB}$ and $\overline{AD}$ are tangent to $\odot C$. Find the value of x.

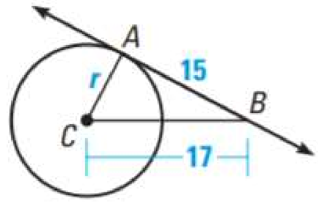
a.


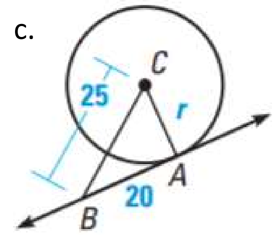
b.


c.


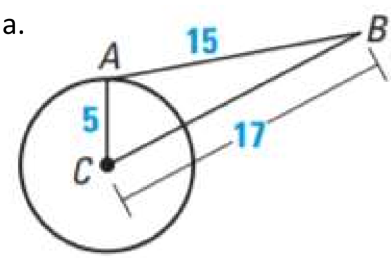
4. $\overleftrightarrow{AB}$ is tangent to $\odot C$. Find the value of r.

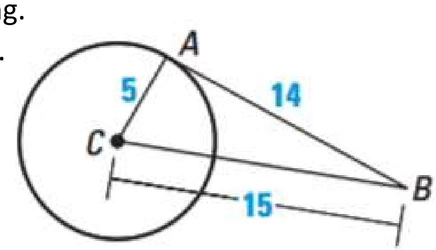
a.


b.


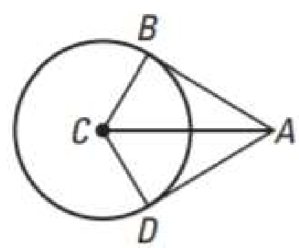
c.


5. Tell whether $\overline{AB}$ is tangent to $\odot C$. Explain your reasoning.

a.


b.


6. $\overline{AB}$ and $\overline{AD}$ are tangent to $\odot C$.



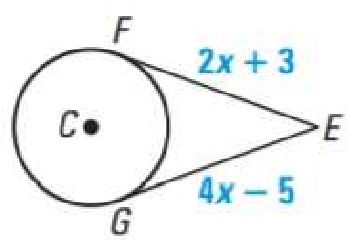
a. Name all congruent segments

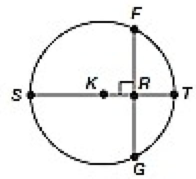
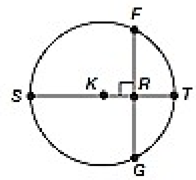
b. Name all congruent angles.

c. Name two congruent triangles.

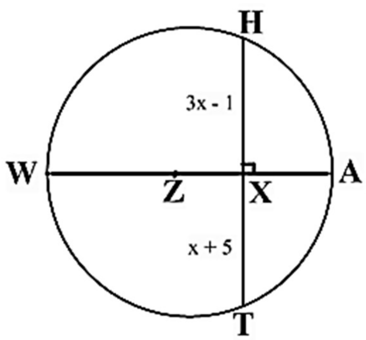
7. MULTIPLE CHOICE: In the diagram below, $\overline{EF}$ and $\overline{EG}$ are tangent to $\odot C$. What is the value of x?

A. -4
B. -1
C. 1
D. 4

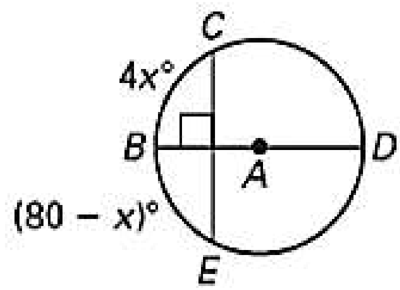


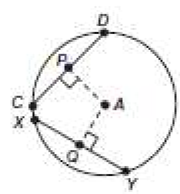
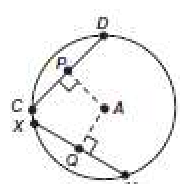
Name	Theorem	Hypothesis	Conclusion
Diameter-Chord Theorem	If a radius or diameter is perpendicular to a chord, then it bisects the chord and its arc.		
Converse of Diameter-Chord Theorem	If a segment is the perpendicular bisector of a chord, then it is the radius or diameter.		

Example: Find the measure of HT. Then find the measure of WA if you know XZ = 6.

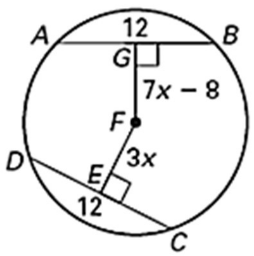


Example: Find the measures of arc CB, BE, and CE.

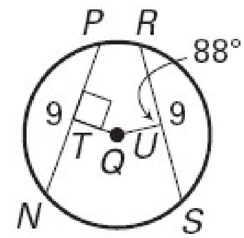


Name	Theorem	Hypothesis	Conclusion
Equidistant Chord Theorem	If two chords are congruent, then they are equidistant from the center.		
Converse of Equidistant Chord Theorem	If two chords are equidistant from the center, then the chords are congruent.		

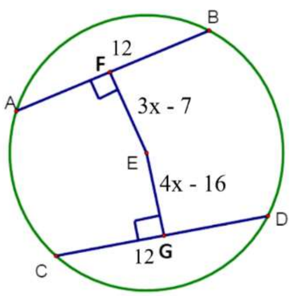
Example: Find EF.



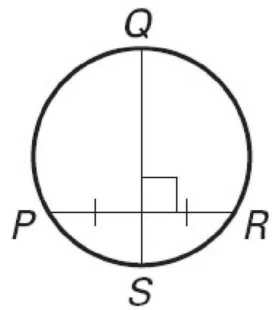
Example: Are segments TQ and UQ congruent?



1. Find the measure of EG.



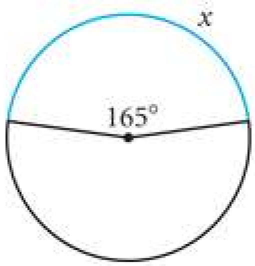
2. Is segment QS a diameter? Explain your reasoning.



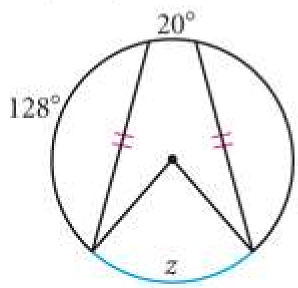
Skills Practice:

Solve for the missing variables.

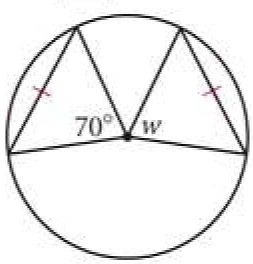
1. $x =$?



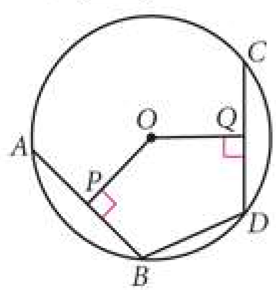
2. $z =$?



3. $w =$?

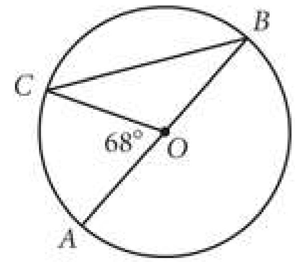


4. $AB = CD$
$PO = 8$ cm
$OQ =$?

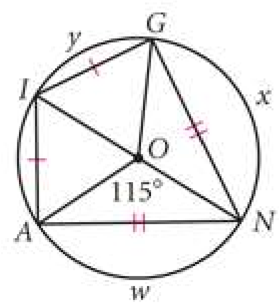


5. $\overline{AB}$ is a diameter.

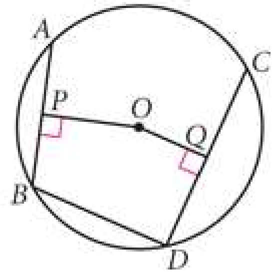
Find $m\widehat{AC}$ and $m\angle B$.



6. *GIAN* is a kite. Find *w*, *x*, and *y*.

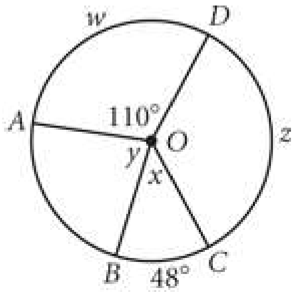


7. $AB = 6$ cm
$OP = 4$ cm
$CD = 8$ cm
$OQ = 3$ cm
$BD = 6$ cm
What is the perimeter of *OPBDQ*?



8. $m\widehat{AC} = 130^\circ$

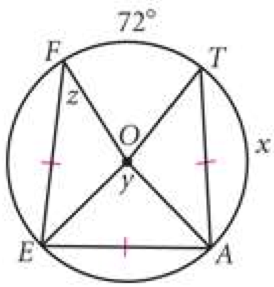
Find w, x, y, and z.



9. $x = \underline{\ ?\ }$

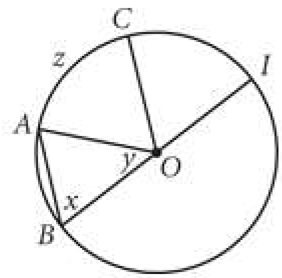
$y = \underline{\ ?\ }$ h

$z = \underline{\ ?\ }$

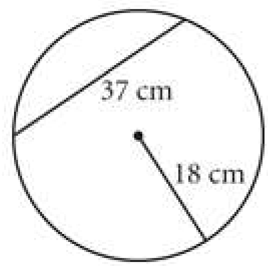


10. $AB || CO, mCI = 66^\circ$

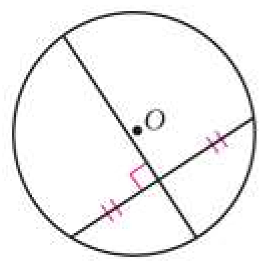
Find x, y, and z.



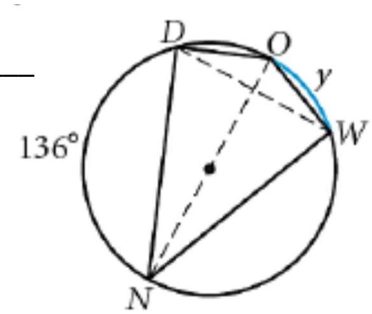
11. What's wrong with this picture?



12. What's wrong with this picture?

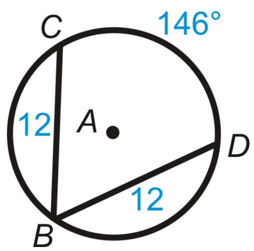


13. y = ________

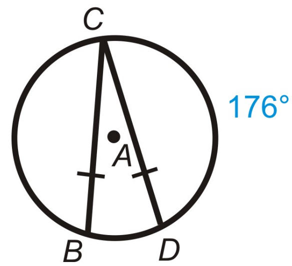


Find the value of the indicated arc in ⊙A.

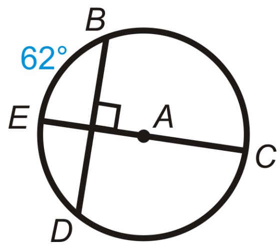
1. $m\widehat{BC}$



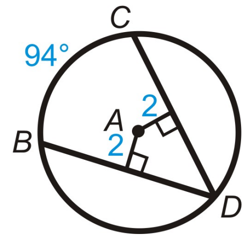
2. $m\widehat{BD}$



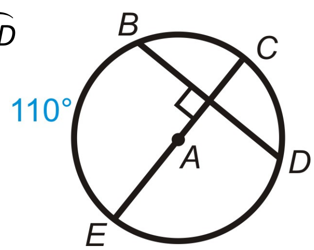
3. $m\widehat{BC}$



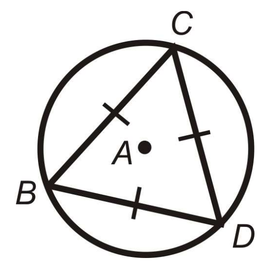
4. $m\widehat{BD}$



5. $m\widehat{BD}$

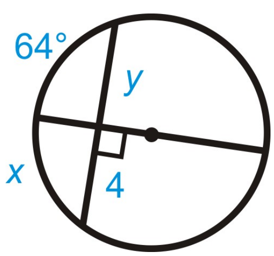


6. $m\widehat{BD}$

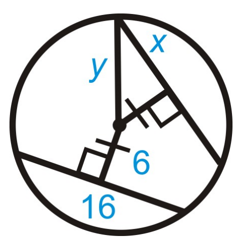


Find the value of *x* and/or *y*.

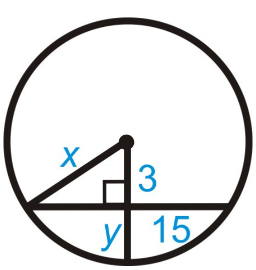
7.



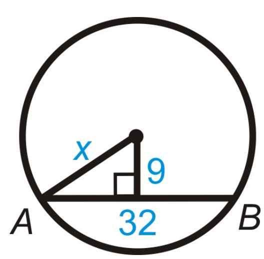
8.



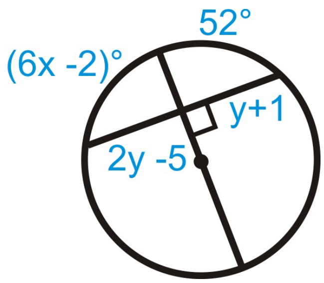
9.



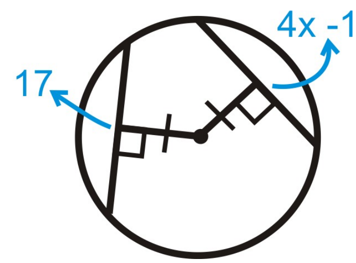
10. AB = 32



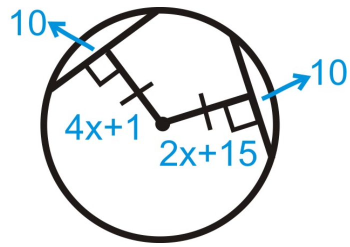
11.



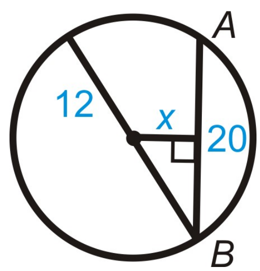
12.



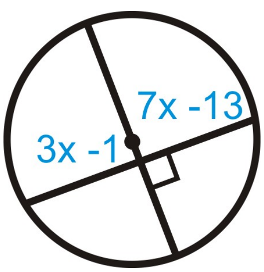
13.



14.

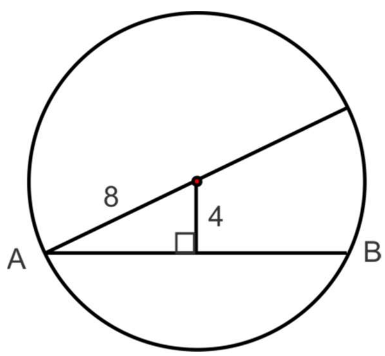


15.

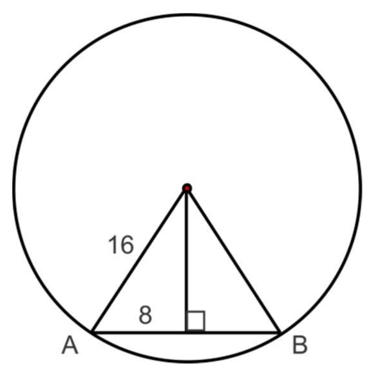


16. Find the measure of $\overset{\frown}{AB}$ in each diagram below.

a.

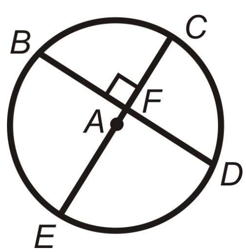


b.

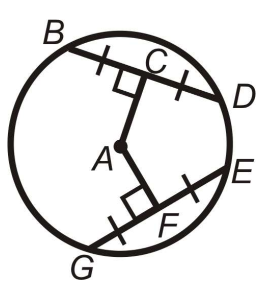


In problems 17-19, what can you conclude about the picture? State a theorem that justifies your answer. You may assume that A is the center of the circle.

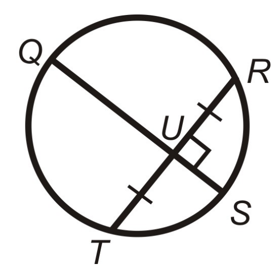
17.



18.

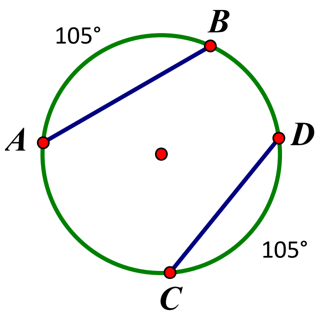


19.

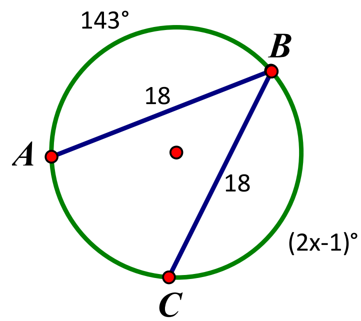


Directions: Use the theorems relating to arcs and chords to find the requested information. Figures are not drawn to scale.

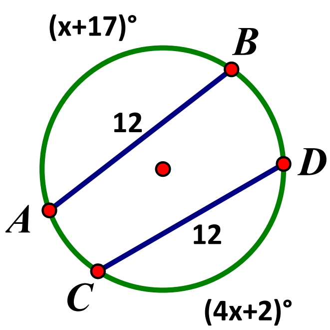
1. $AB = 10x - 1$ and $CD = 2x + 23$; x = ________



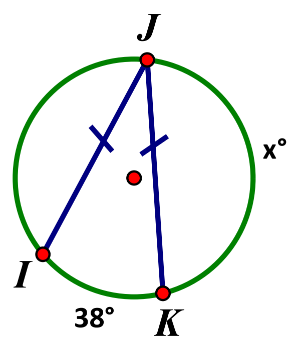
2. x = ________



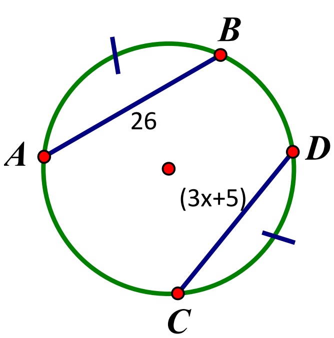
3. x = ________



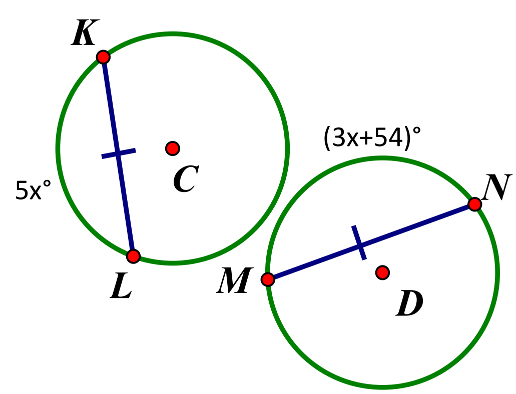
4. $x°$ = ________



5. x = ________



6. $\odot C \cong \odot D$; x = ______

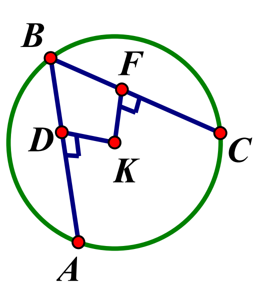


7. In $\odot K$, $\overline{AB} \cong \overline{BC}$.

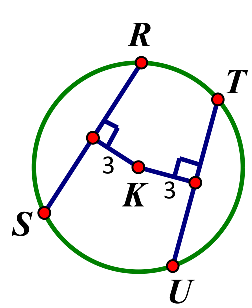
$DK = 5x + 6$

$FK = 2x + 21$

x = ________

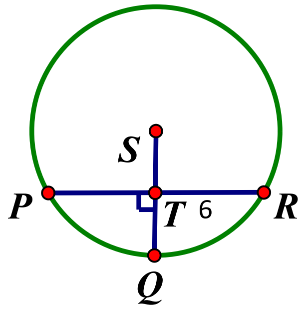


8. In $\odot K$, $SR = 24, UT = 3x$ x = ______



9. In $\odot S$, $m\widehat{PR} = 98°$ and $TR = 6$

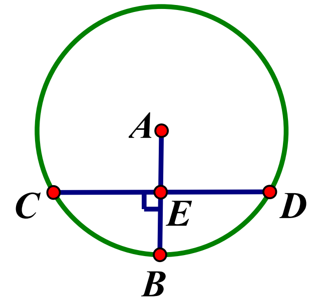
$m\widehat{PQ}$ = _______; PR = ________



*The radius is perpendicular to a chord, so it bisects the chord and the arc.

10. In $\odot A$, $radius = 14$ and $CD = 22$

Find CE and EB. Round to 2 decimals.



CE = _______

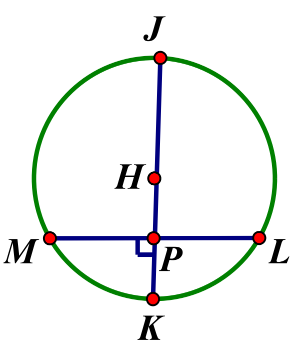
EB = _______

*To find CE: The radius bisects chord $\overline{CD}$

*To find EB: $\overline{AB}$ is the radius of the circle, but so is $\overline{AC}$ or $\overline{AD}$. Create a right triangle to use the Pythagorean Theorem to find AE. Then use

11. In $\odot H$, $diameter = 18$, $LM = 12$, and $m\widehat{LM} = 84°$.
Find each measure.
Round to the nearest hundredth if necessary.

*Use the ideas from #9 and #10 to solve this problem.

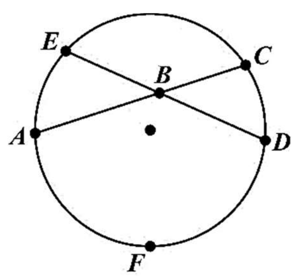


$m\widehat{LK}$ = ______

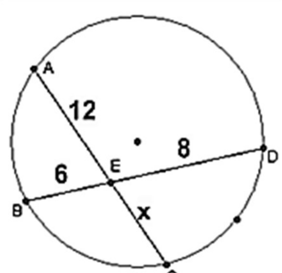
HP = ______

PK = ______

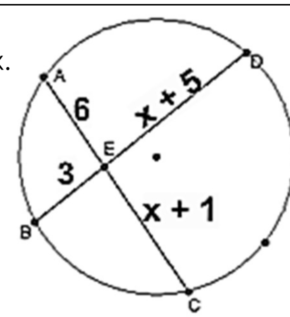
Segment Lengths (In and Out of a Circle)

Name	Theorem	Hypothesis	Conclusion
Segment Chord Theorem	If two chords in a circle interest, then the product of the lengths of the segments of one chord is equal to the product of the lengths of the segments of the second chord.		**POP = POP** **Product of Pieces = Product of Pieces**

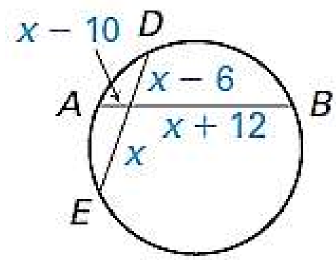
Example: Find x.



Example: Find x.

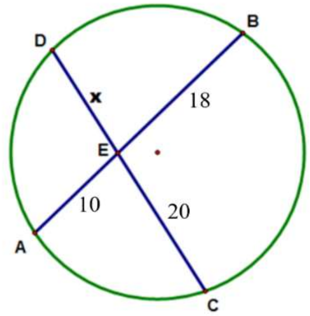


Example: Find x.

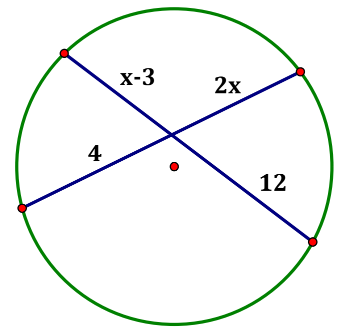


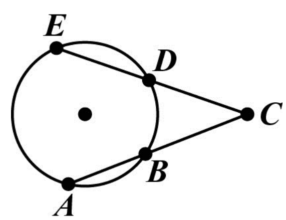
Try:

1. Find the value of x.

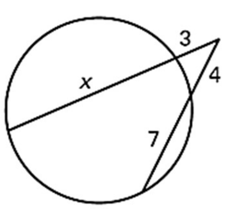


2. Find the value of x.

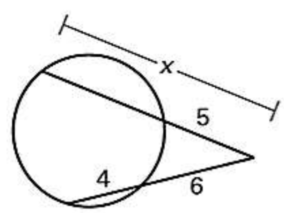


Secant Segment Theorem	If two secant segments intersect in the exterior of a circle, then the product of the lengths of the secant segment and its external secant segment is equal to the product of the lengths of the second secant segment and its external secant segment.		**OW = OW** **Outside * Whole = Outside * Whole**

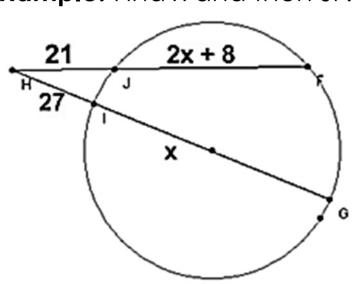
Example: Find x.



Example: Find x.

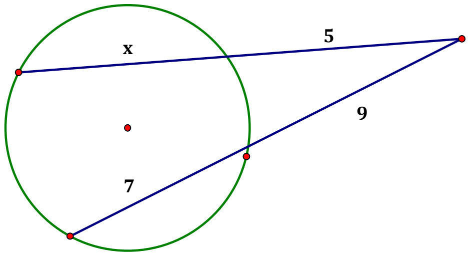


Example: Find x and then JF.

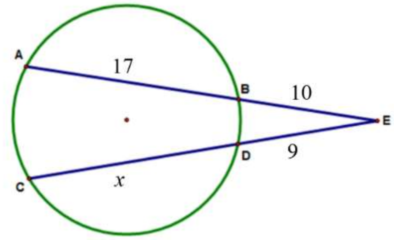


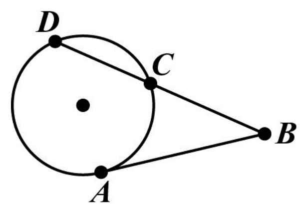
Try:

1. Find the value of x.

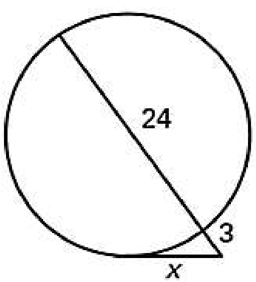


2. Find he value of x.

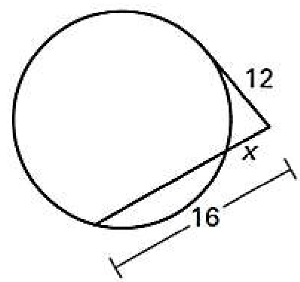


Secant Tangent Theorem	If a tangent and secant intersect in the exterior of a circle, then the product of the lengths of the secant segment and its external secant segment is equal to the square of the length of the tangent segment.		**OW = OW** **Outside * Whole = Outside * Whole**

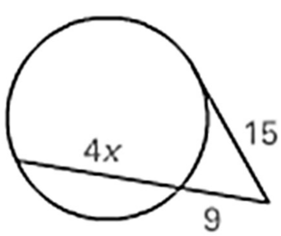
Example: Find x.



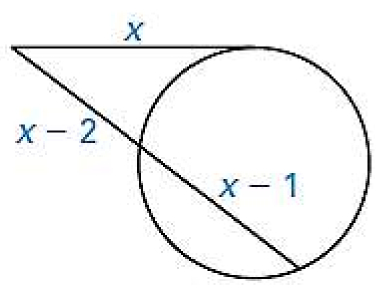
Example: Find x.



Example: Find x.

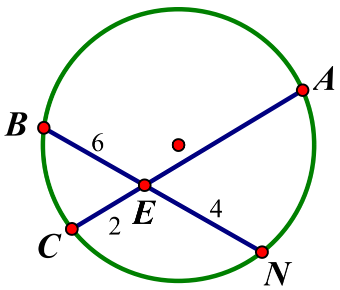


Example: Find all possible values of x.

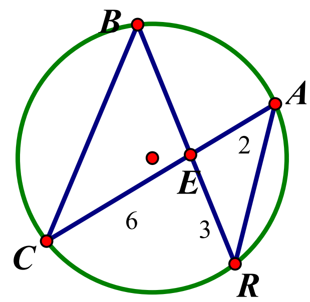


Guided Practice: Find the missing indicated segment in each of the following examples.

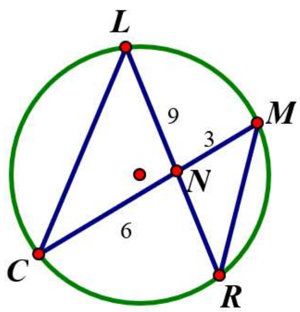
1. Find AE.



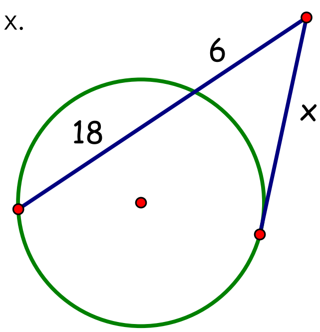
2. Find BE.



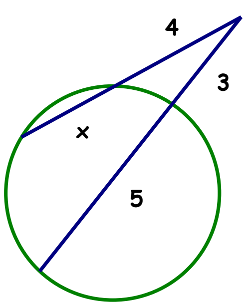
3. Find NR.



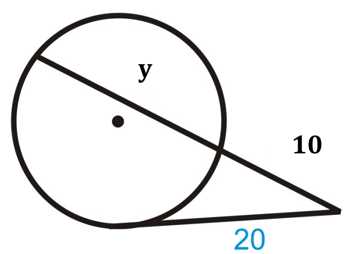
4. Find x.



5. Find x.



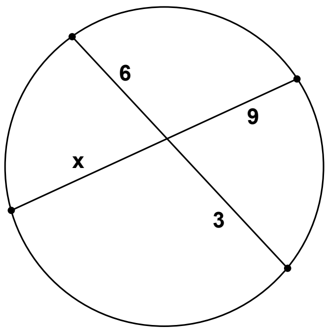
6.



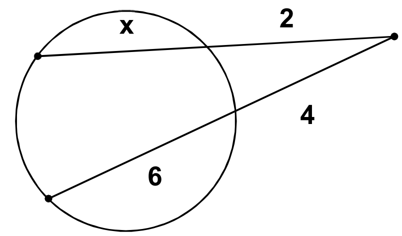
Skills Practice

Find the value of the missing variable.

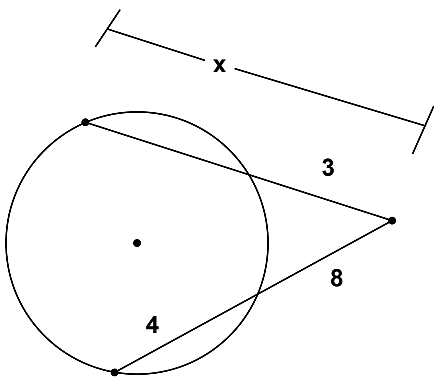
1.



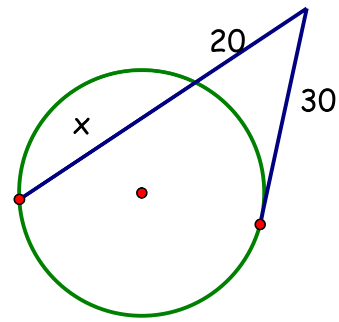
2.



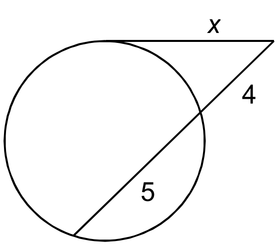
3.



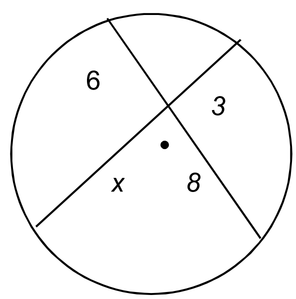
4.



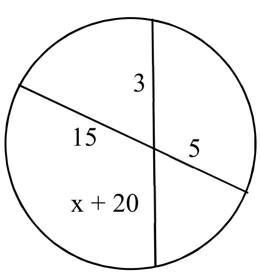
5.



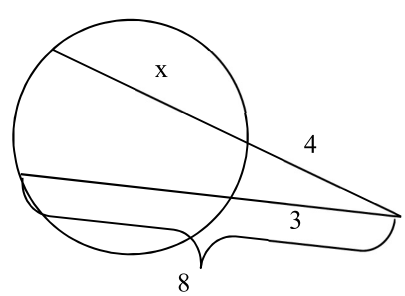
6.



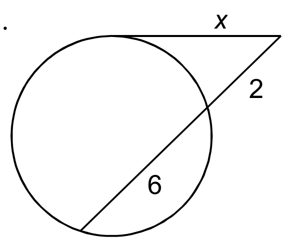
7.



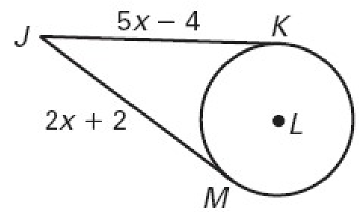
8.



9.

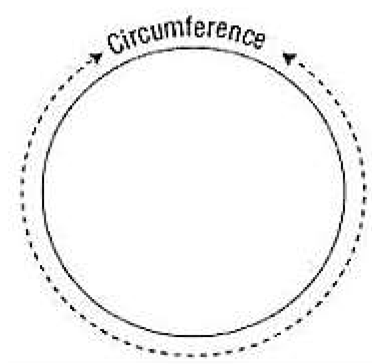


10.



Arc Length

In 7th grade, you learned how to calculate the circumference of a circle. You also learned that the circumference of a circle divided by the diameter is equal to pi. The circumference of a circle is the distance around the circle.



Circumference
$C = 2\pi r$ or $C = \pi d$

Practice reviewing how to calculate the circumference or radius/diameter of a circle below. Leave your answers in terms of pi. Find the circumference, radius, or diameter.

A. r = 6 ft B. d = 15 in C. $C = 16\pi$ cm D. $C = 40\pi$ m

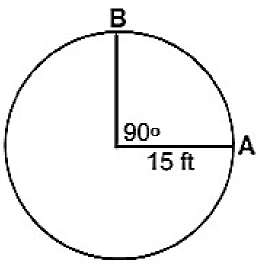
Calculating Arc Length

Arc Length is a fraction of the circle's circumference and is measured in linear units.

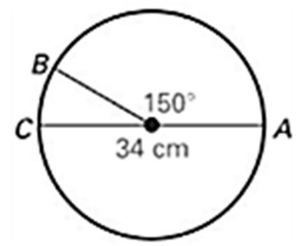
Arc Length

$$2\pi r \bullet \frac{\theta}{360} = \frac{2\pi r\theta}{360}$$ **, where θ is the central angle (or intercepted arc measure**

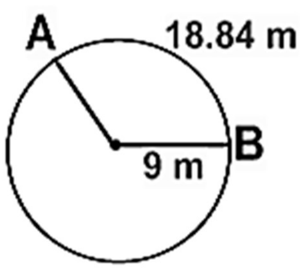
Example: Find the length of arc BA.



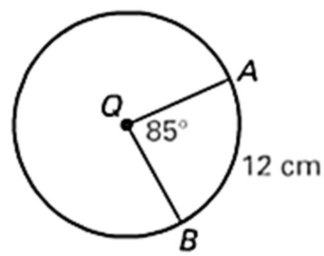
Example: Find the length of arc BC.



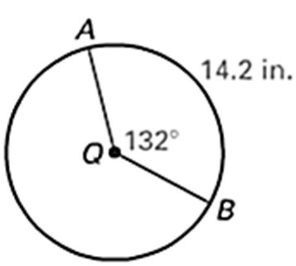
Example: Find the measure of arc AB.



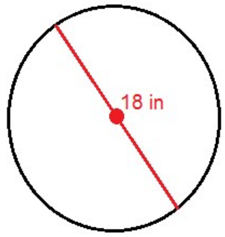
Example: Find the circumference of Circle Q.



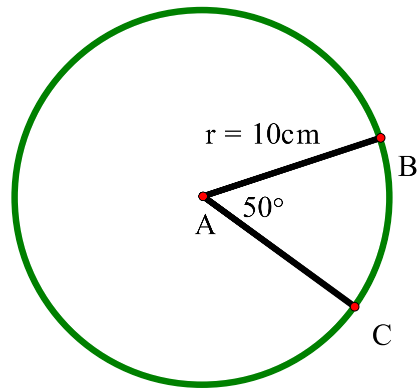
Example: Find the radius of Circle Q.



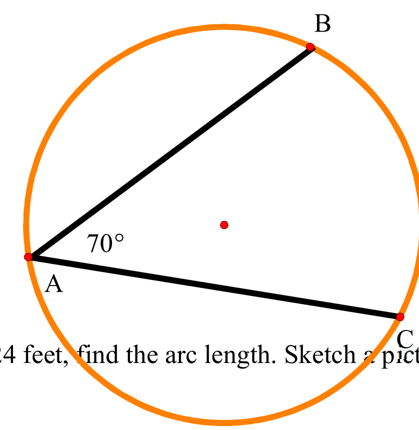
Use the formulas to answer the questions below. Be sure to leave all answers in terms of pi.
EXAMPLE 1: Find the circumference of the circle.



Example 2: Use the diagram of the circle to find the arc length of BC.

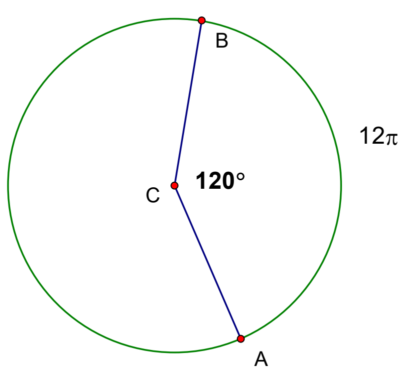


Example 3: Use the diagram of the circle to find the arc length of BC with a radius of 4 inches.



Example 4: If a central angle measures 80° and the diameter of the circle measures 24 feet, find the arc length. Sketch a picture to help you solve the problem.

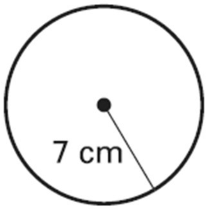
Example 5: Use the formula that you have developed for arc length and find the **circumference** of the circle.



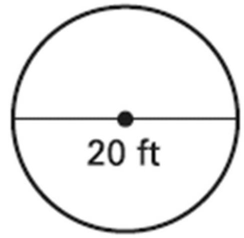
Skills Practice: Calculating Arc Length and Circumference

Use the diagram to find the indicated measure. Leave answers in term of pi.

1. Find the circumference.



2. Find the circumference.



3. Find the radius. **Find the indicated measure.**

a. The exact radius of a circle with circumference 36 meters

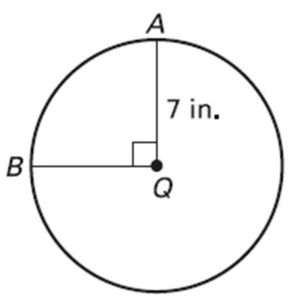
b. The exact diameter of a circle with circumference 29 feet

c. The exact circumference of a circle with diameter 26 inches

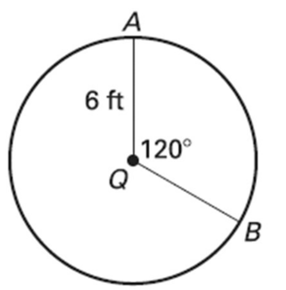
d. The exact circumference of a circle with radius 15 centimeters

4. Find the length of $\overset{\frown}{AB}$.

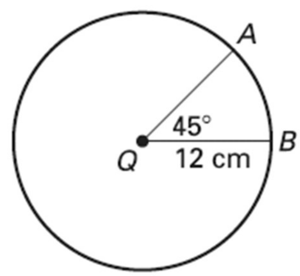
a.



b.



c.



5. In ⊙ *D* shown below, $\angle ADC \cong \angle BDC$. Find the indicated measure

a. $m\overset{\frown}{CB}$

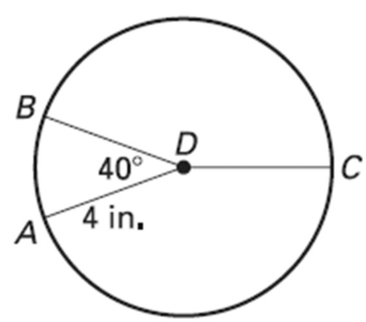
b. $m\overset{\frown}{ACB}$

e. $m\overset{\frown}{BAC}$

c. Length of $\overset{\frown}{CB}$

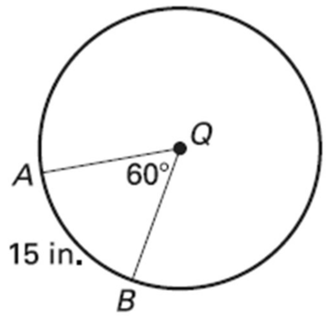
f. Length of $\overset{\frown}{ACB}$

d. Length of $\overset{\frown}{ABC}$

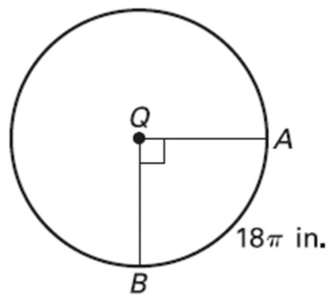


6. **Find the indicated measure.**

a. The radius of circle Q

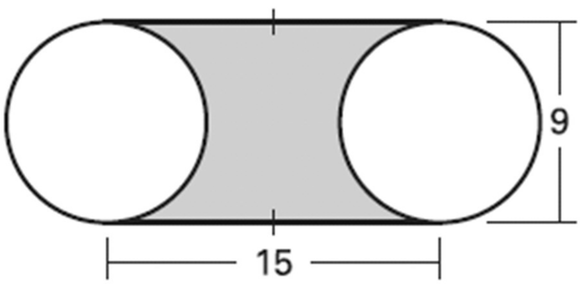


b. Circumference of ⊙ *Q and* Radius of ⊙ Q



Find the perimeter of the region. Round to the nearest hundredth.

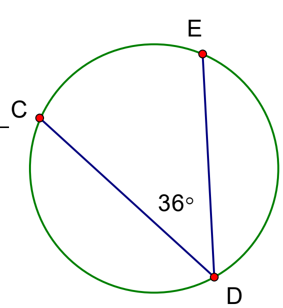
7.



8. **Birthday Cake** A birthday cake is sliced into 8 equal pieces. The arc length of one piece of cake is 6.28 inches as shown. Find the diameter of the cake.

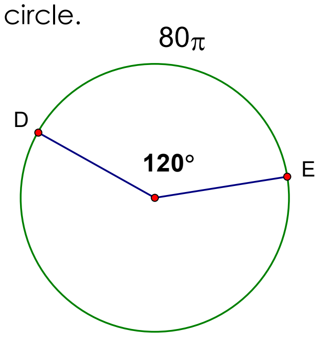
9. Radius = 5 in

Length of Arc CE = _______



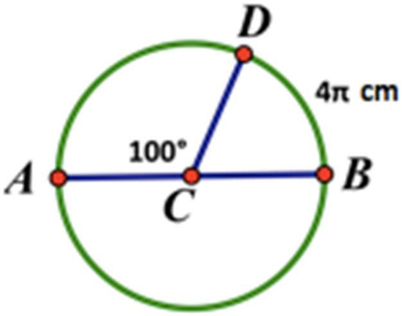
10. Find the radius of the circle.

r = ___________

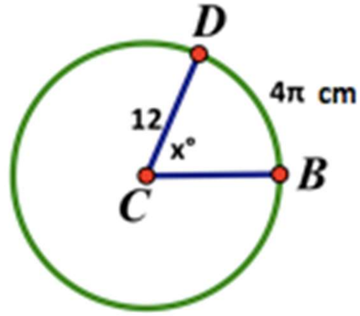


For #11-13, solve for the requested variable. C is the center of each circle.

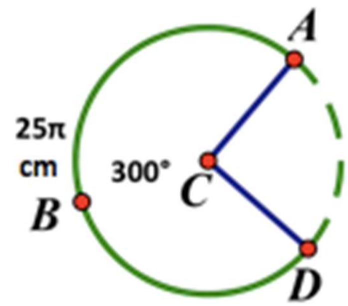
11. r = _______



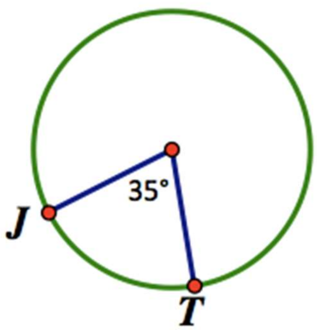
12. x° = ________



13. d = ______

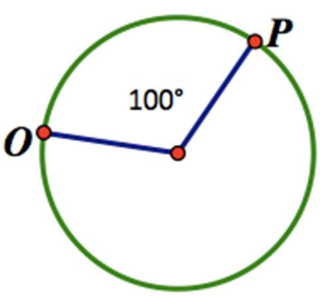


14. Circumference = 10 m; Find the arc length of $\overset{\frown}{JT}$ = ______



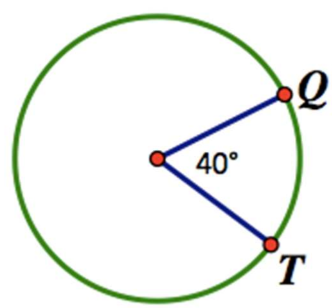
15. The arc length of $\overset{\frown}{OP} = 10\pi$ inches;

r = _______

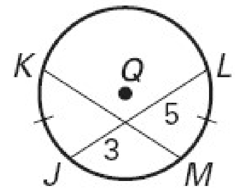
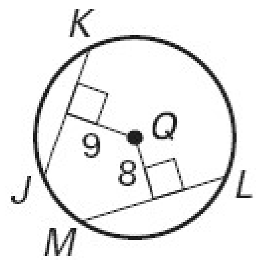
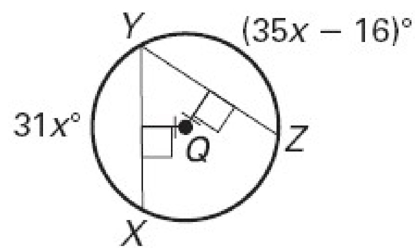
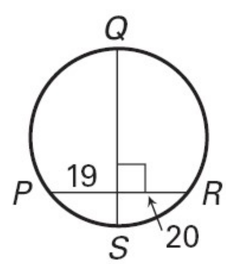


16. The arc length of $\overset{\frown}{QT} = 22$ cm.;

d = _______ (to the tenth)

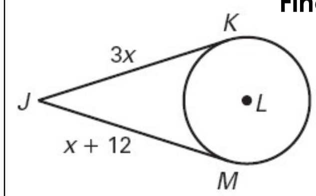
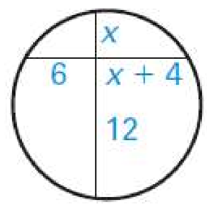
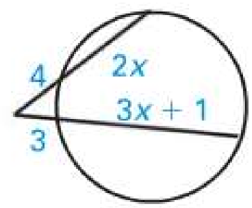
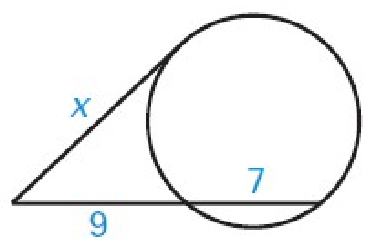


<u>**Theorems with Chords and Arcs Graphic Organizer**</u>

Theorem	**Example**
If two chords are congruent then their arcs are congruent	**Find the length of KM.** 
Two chords are congruent if they are equidistant from the center of the circle	**Are $\overline{JK}$ and $\overline{ML}$ congruent?** 
Two chords are congruent if and only if they are equidistant from the center of the circle.	**Find the measure of YX.** 
To be a diameter the chord must be a perpendicular bisector of another chord.	**Is $\overline{QS}$ a diameter? Why or why not?** 
Pythagorean Theorem.	**A chord in a circle is 18 cm long and is 5 cm from the center of the circle. What is the length of the radius of the circle?**

The Segment Theorems Graphic Organizer

Let's summarize the theorems relating to tangents, chords, and secants. Use the information from the previous task to complete the graphic organizer.

Picture	Type	Theorem	Example
			Find JK 
			Solve for x. 
			Find the value of x. 
			Find the value of x. 
			Find the value of a.