

Triangle Congruence Theorems

Key

Side-Side-Side (SSS) Congruence Postulate

If three sides of one triangle are congruent to three sides of another triangle, then the triangles are congruent.

$QR \cong TU$, $RP \cong US$, and $PQ \cong ST$, so $\triangle PQR \cong \triangle STU$.

Side-Angle-Side (SAS) Congruence Postulate

If two sides and the included angle of one triangle are congruent to two sides and the included angle of another triangle, then the triangles are congruent.

$\angle K$ is the included angle of \overline{HK} and \overline{KJ} .

$\angle N$ is the included angle of \overline{LN} and \overline{NM} .

$\triangle HJK \cong \triangle LMN$

Angle-Side-Angle (ASA) Congruence Postulate

If two angles and the included side of one triangle are congruent to two angles and the included side of another triangle, then the triangles are congruent.

\overline{AC} is the included side of $\angle A$ and $\angle C$.

\overline{DF} is the included side of $\angle D$ and $\angle F$.

$\triangle ABC \cong \triangle DEF$

Angle-Angle-Side (AAS) Congruence Theorem

If two angles and a nonincluded side of one triangle are congruent to the corresponding angles and nonincluded side of another triangle, then the triangles are congruent.

\overline{FH} is a nonincluded side of $\angle F$ and $\angle G$.

\overline{JL} is a nonincluded side of $\angle J$ and $\angle K$.

$\triangle FGH \cong \triangle JKL$

Hypotenuse-Leg (HL) Congruence Theorem

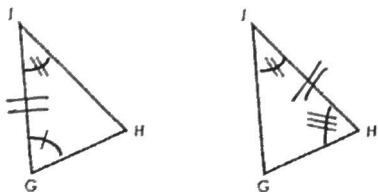
If the hypotenuse and a leg of a right triangle are congruent to the hypotenuse and a leg of another right triangle, then the triangles are congruent.

$\triangle JKL \cong \triangle MNP$

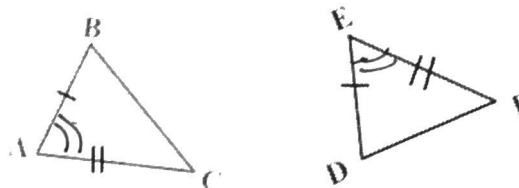
Included Side
The side between two angles

Included Angle
The angle between two sides

Practice: Mark the included side in each triangle



Practice: Mark the included angle in each triangle.



Definition of Congruence

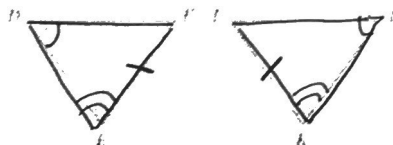
Two triangles are congruent to each other if and only if their corresponding angles and sides are congruent. Corresponding parts of triangles are the parts of the congruent triangles that "match."

Example: If $\triangle RST \cong \triangle XYZ$, identify all pairs of congruent corresponding parts. Draw a picture and label the congruent angles and sides.

<u>Angles</u>	<u>Sides</u>
$\angle R \cong \angle X$	$\overline{RS} \cong \overline{XY}$
$\angle S \cong \angle Y$	$\overline{ST} \cong \overline{YZ}$
$\angle T \cong \angle Z$	$\overline{TR} \cong \overline{ZX}$

What additional information is needed to prove the following triangles are congruent by the stated theorem?

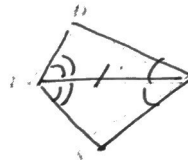
a. AAS



b. SAS



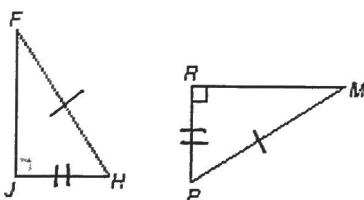
c. ASA



Practice: Mark the appropriate sides and angles to make each congruence statement true by the stated congruence theorem.

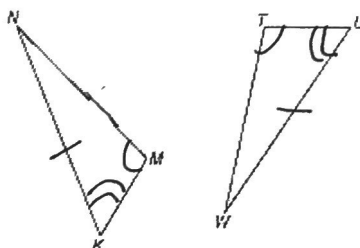
a. HL

$\triangle JFH \cong \triangle RMP$



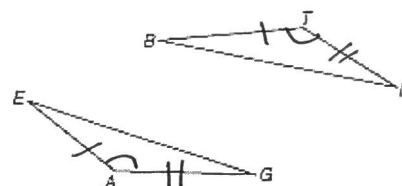
b. AAS

$\triangle NMK \cong \triangle WTU$



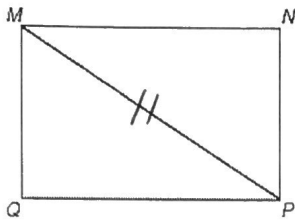
c. SAS

$\triangle EGA \cong \triangle NBT$



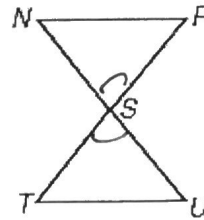
Markings You Are Allowed to Add

Share a Side



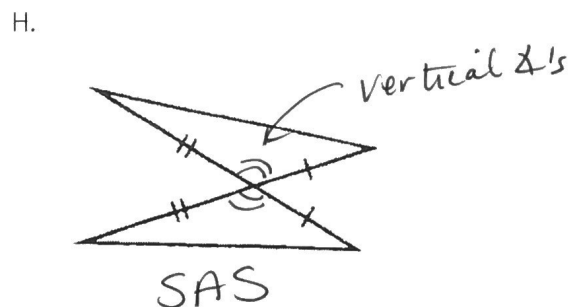
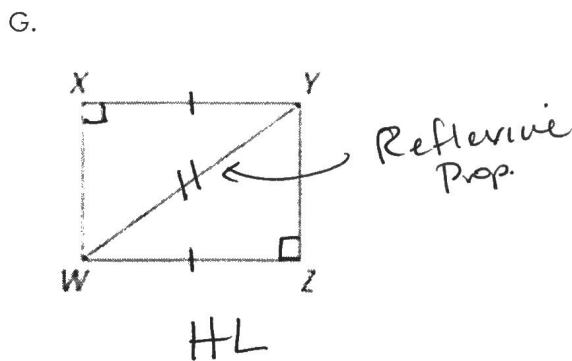
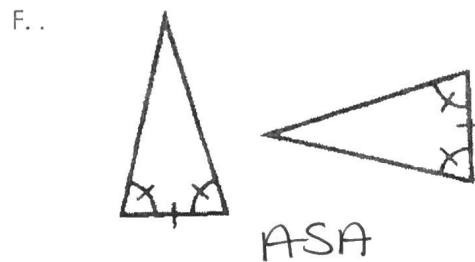
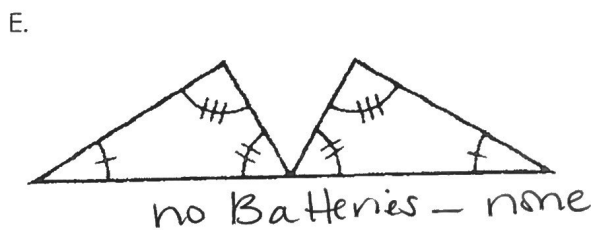
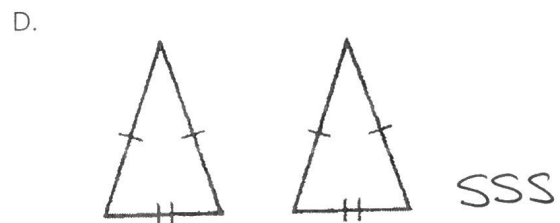
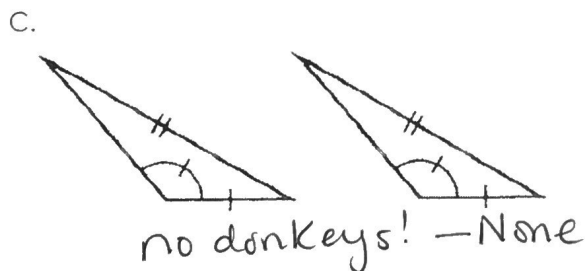
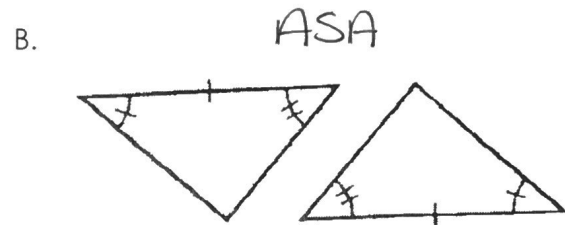
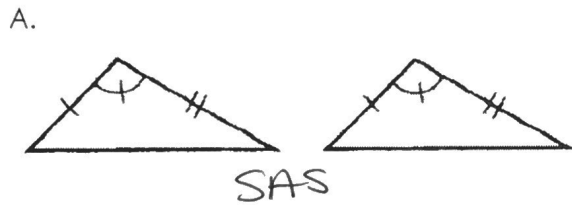
Reason: Reflexive Property

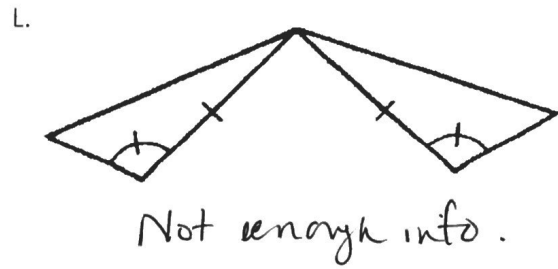
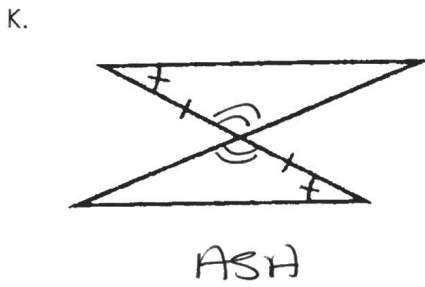
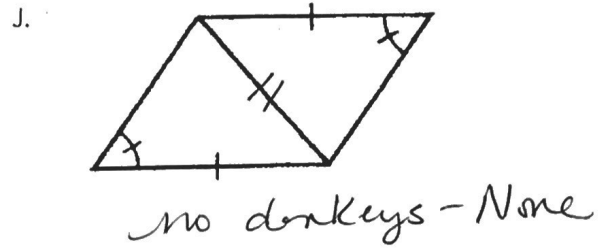
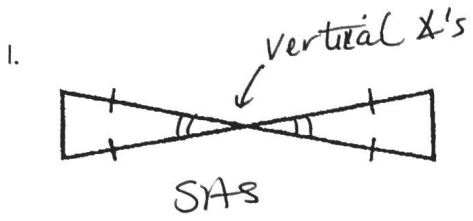
Vertical Angles



Reason: Vertical Angles are Congruent

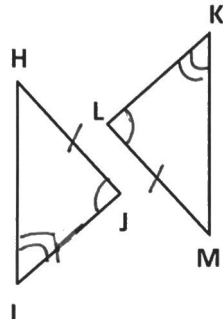
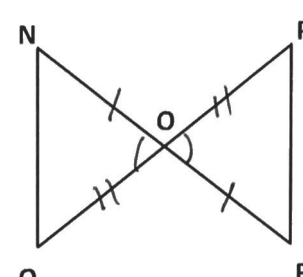
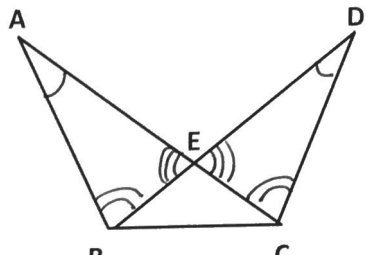
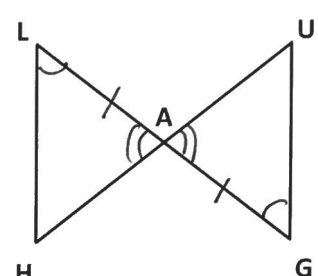
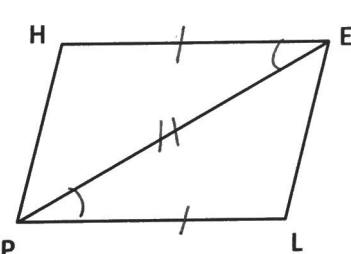
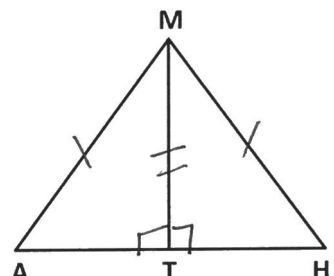
Practice: Determine whether there is enough information to conclude if the triangles are congruent. If so, state the congruence theorem. If not, write not enough information.





Congruence Statements Revisited
GGA

For each problem below, a) mark the diagram with the given information, b) add any additional congruence information that you know to be true "freebies", then c) decide whether you can conclude the triangles are congruent. If you can, state the congruence rule that supports your conclusion and write a triangle congruence statement. If you can't, write "not congruent".

<p>1. Given: $\overline{HJ} \cong \overline{LM}$ $\angle J \cong \angle L$ $m\angle I = m\angle K$</p>  <p>Congruence Rule: <u>AAS</u> Statement: $\triangle HIJ \cong \triangle LMK$</p>	<p>2. Given: $\overline{NO} \cong \overline{OR}$ $\overline{QO} \cong \overline{OP}$</p>  <p>Congruence Rule: <u>SAS</u> Statement: $\triangle NOQ \cong \triangle ROP$</p>	<p>3. Given: $\angle A \cong \angle D$ $\angle ABE \cong \angle DCE$</p>  <p>Congruence Rule: <u>none!</u> Statement: $\triangle ABE \cong$ _____</p>
<p>4. Given: $\angle A = \angle G$ $\angle L \cong \angle G$</p>  <p>Congruence Rule: <u>ASA</u> Statement: $\triangle LAH \cong \triangle GAU$</p>	<p>5. Given: $\angle HEP \cong \angle LPE$ $\overline{HE} \cong \overline{PL}$</p>  <p>Congruence Rule: <u>SAS</u> Statement: $\triangle HEP \cong \triangle LPE$</p>	<p>6. Given: $\overline{MT} \perp \overline{AH}$ $\overline{MA} \cong \overline{MH}$</p>  <p>Congruence Rule: <u>HL</u> Statement: $\triangle MAT \cong \triangle MHT$</p>

Use the triangle congruence statements below to identify all congruent angles and all congruent sides.

<p>7. $\triangle BOY \cong \triangle HAT$</p> <p>$\angle B \cong \angle H$ $\overline{BO} \cong \overline{HA}$ $\angle O \cong \angle A$ $\overline{OY} \cong \overline{AT}$ $\angle Y \cong \angle T$ $\overline{BY} \cong \overline{HT}$</p>	<p>8. $\triangle ART \cong \triangle WIN$</p> <p>$\angle A \cong \angle W$ $\overline{AR} \cong \overline{WI}$ $\angle R \cong \angle I$ $\overline{RT} \cong \overline{IN}$ $\angle T \cong \angle N$ $\overline{TA} \cong \overline{NW}$</p>
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