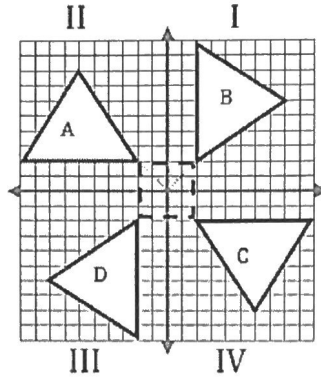


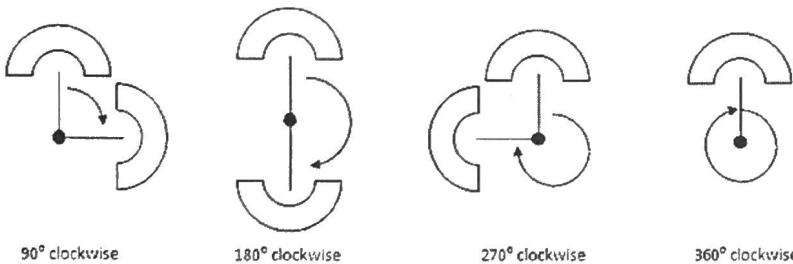
## Day 2 – Rotations, Symmetry, and Multiple Transformations

A **rotation** is a circular movement around a central point that stays fixed and everything else moves around that point in a circle. A rotation maintains size and shape; therefore, it is our third type of rigid transformation.



When we rotate our figures around a fixed point, we classify our rotation by direction and degree of rotation.

### Degrees of Rotation



### Direction of Rotation



It is important to understand that some rotations are the same depending on the degree and direction of the rotation. Most of the time, rotations are given using counterclockwise direction. Here are equivalent rotations:

$90^\circ$  Counterclockwise =  $270^\circ$  Clockwise       $90^\circ$  Clockwise =  $270^\circ$  Counterclockwise

$180^\circ$  Counterclockwise =  $180^\circ$  Clockwise

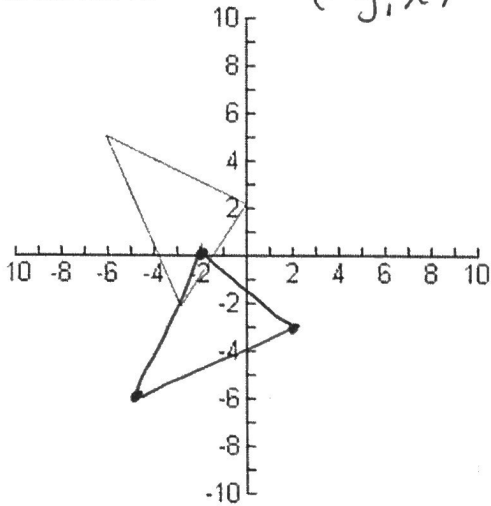
### Rules for Rotations

| $90^\circ$ CCW / $270^\circ$ CW | $180^\circ$ CCW / $180^\circ$ CW | $90^\circ$ CW / $270^\circ$ CCW |
|---------------------------------|----------------------------------|---------------------------------|
| $(x, y) \rightarrow (-y, x)$    | $(x, y) \rightarrow (-x, -y)$    | $(x, y) \rightarrow (y, -x)$    |
|                                 |                                  |                                 |

Practice with Rotations

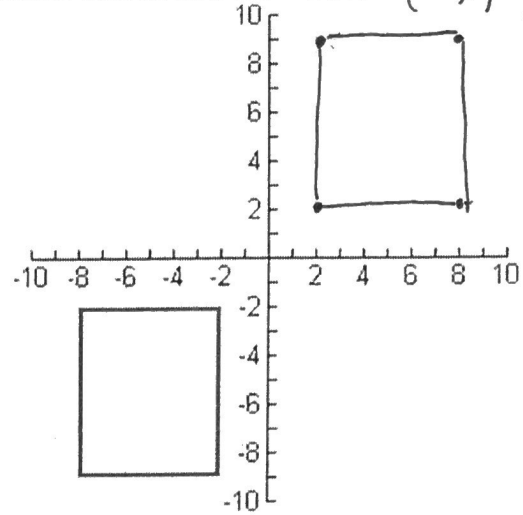
a. Rotate  $90^\circ$

$(-y, x)$



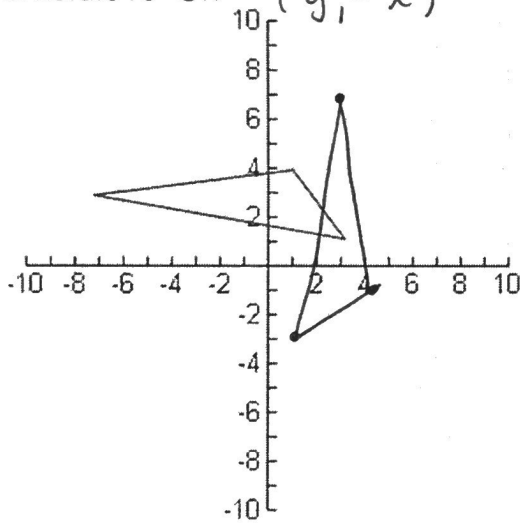
bb. Rotate 180. Rotate  $180^\circ$  CCW

$(-x, -y)$



c. Rotate  $90^\circ$  CW

$(y, -x)$



d. The line segment with endpoints  $(7, -8)$  and  $(-3, -5)$  are rotated  $90^\circ$  CW. What are its new endpoints?

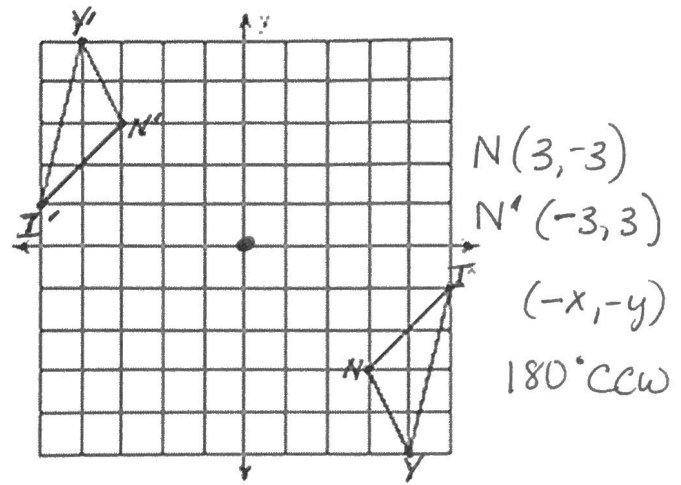
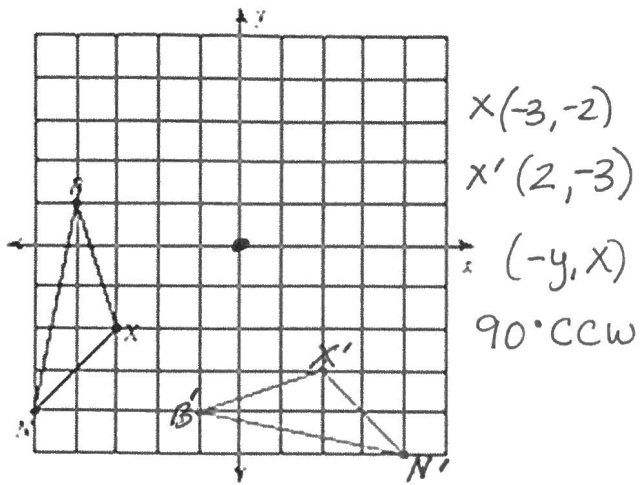
$(y, -x)$

$(7, -8) \rightarrow (-8, -7)$

$(-3, -5) \rightarrow (-5, 3)$

e. Describe the rotations:

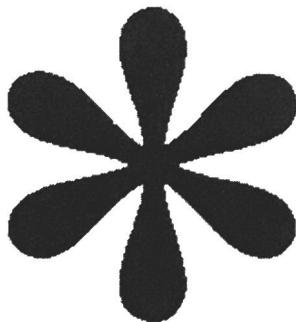




**Rotational Symmetry**

A figure has **rotational symmetry** if there is a center point around which the object is turned a certain number of degrees and the object looks the same. The degree of rotational symmetry that an object has is called its **order**. The order of rotational symmetry that an object has is the number of times that it fits onto itself during a full rotation of 360 degrees. To determine the **angle of rotation**, divide 360 degrees by its order.

**Example:** Determine the order and angle of rotation:

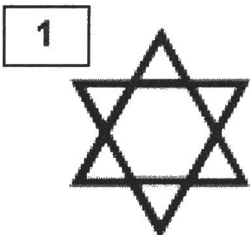


$$\frac{360}{6} = 60^\circ$$

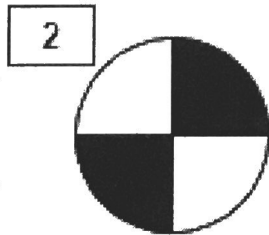


$$\frac{360}{5} = 72^\circ$$

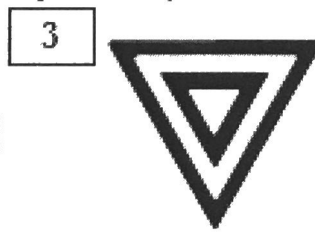
**Practice:** For the following figures, name the order and degrees of rotation:



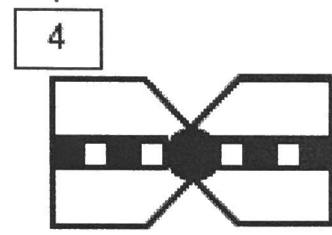
60°



90°



120°

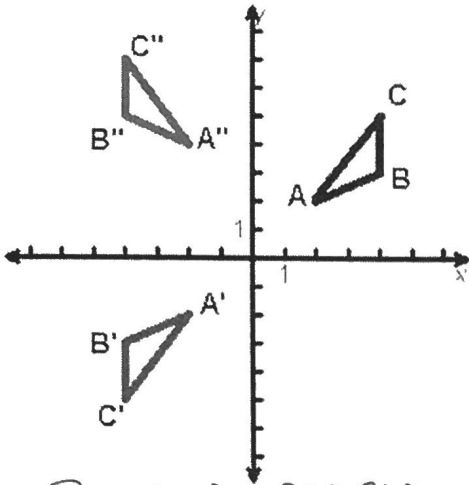


180°

Combined Transformations

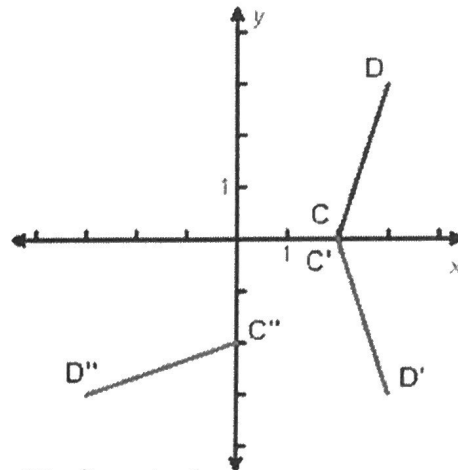
**Practice:** Describe the transformations that occurred from the pre-image to each image (in order).

A.



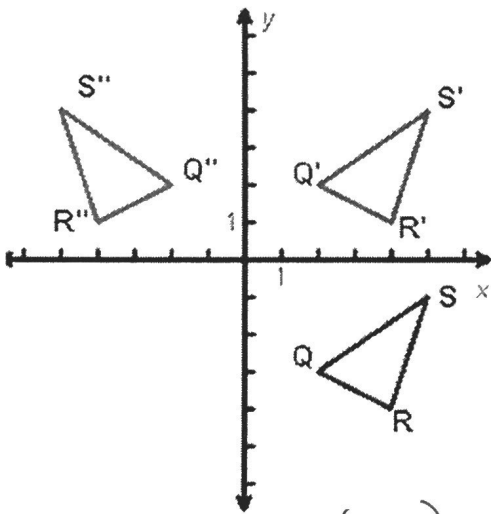
1. Rotation  $180^\circ$  CW
2. Reflection over  $y=1$

B.

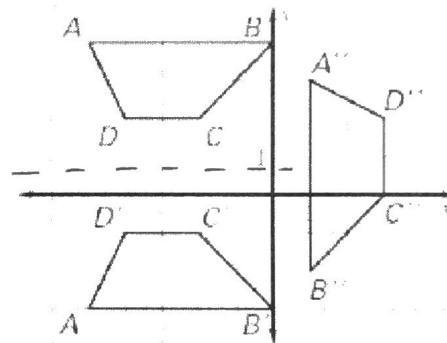


1. Reflection over  $x$ -axis ( $y=0$ )
2. Rotation  $90^\circ$  CW

C.



1. Translation  $(x, y) \rightarrow (x, y+5)$
2. Reflection over  $y$ -axis ( $x=0$ )

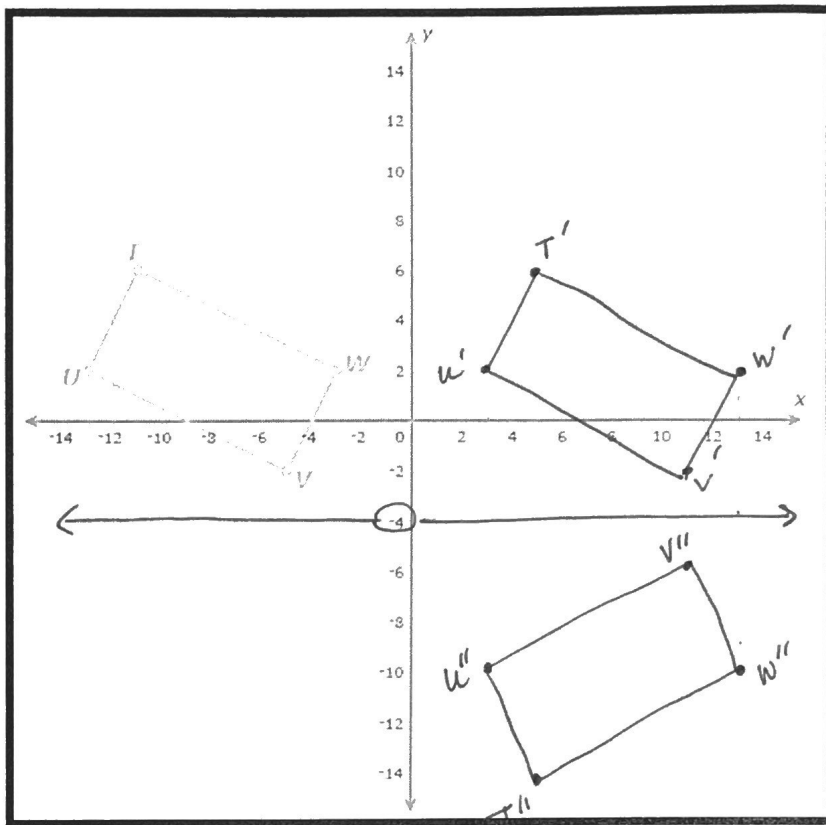


1. Reflection over  $y = \frac{1}{2}$
  2. Rotation  $90^\circ$  CW @ B \*
  3. Translate  $(x+1, y+1)$  B \*
- $A'(-5, -3) \quad A''(1, 3)$

**Perform the following transformations:**

a. Translation  $(x, y) \rightarrow (x + 16, y)$ :

b. Reflection across  $y = -4$ :

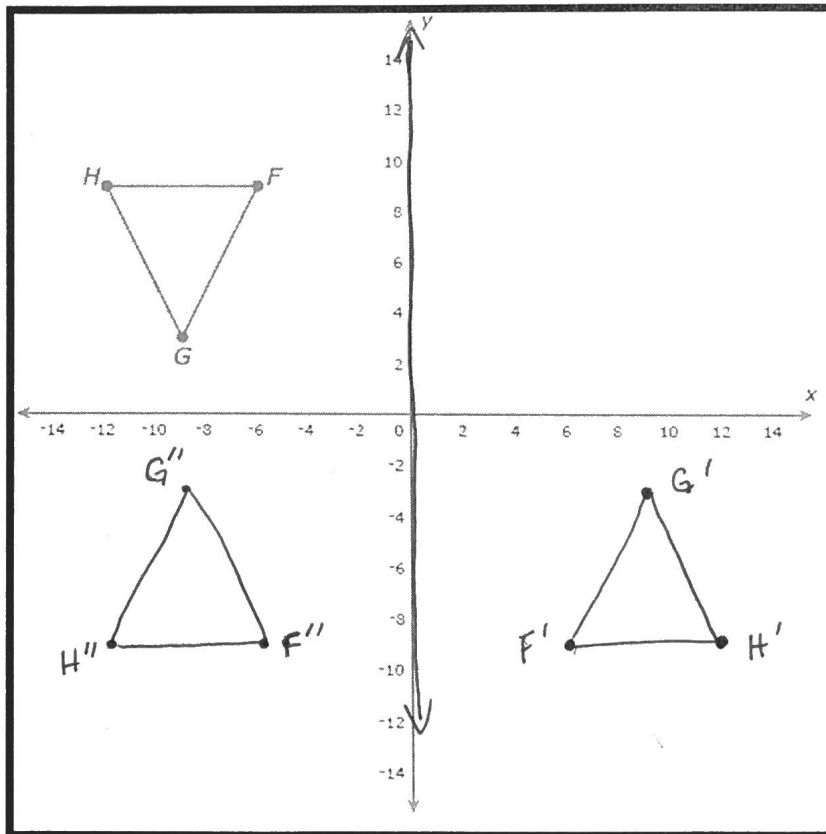


**Perform the following transformations:**

a. Rotation  $180^\circ$  CW around the origin:

$$(-x, -y)$$

b. Reflection across the y-axis:



**Perform the following transformations:**

a. Rotation 90° CCW around the origin:

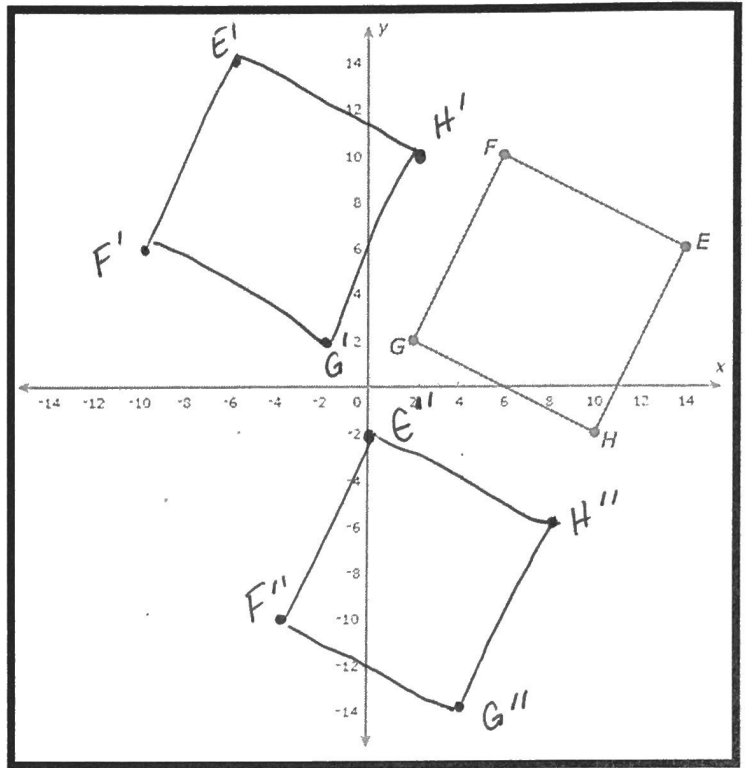
$$G (2, 2) \rightarrow (-2, 2)$$

$$F (6, 10) \rightarrow (-10, 6)$$

$$E (14, 6) \rightarrow (-6, 14)$$

$$H (10, -2) \rightarrow (2, 10)$$

b. Translation  $(x, y) \rightarrow (x + 6, y - 16)$



**Summary of Transformation Rules**

| Transformation          | Rules   | Examples |
|-------------------------|---|----------|
| Translations<br>"slide" | $(x, y) \rightarrow (x \pm a, y \pm b)$<br>a: horizontal slide (+ right, - left)<br>b: vertical slide (+ up, - down)  |          |
| Reflections<br>"flip"   | x-axis: $(x, y) \rightarrow (x, -y)$<br>y-axis: $(x, y) \rightarrow (-x, y)$<br>$y = x$ : $(x, y) \rightarrow (y, x)$<br>$y = -x$ : $(x, y) \rightarrow (-y, -x)$   |          |
| Rotations<br>"turn"     | $90^\circ \text{ CW} = 270^\circ \text{ CCW}: (x, y) \rightarrow (y, -x)$<br>$180^\circ \text{ CW} = 180^\circ \text{ CCW}: (x, y) \rightarrow (-x, -y)$<br>$90^\circ \text{ CCW} = 270^\circ \text{ CW}: (x, y) \rightarrow (-y, x)$ |          |