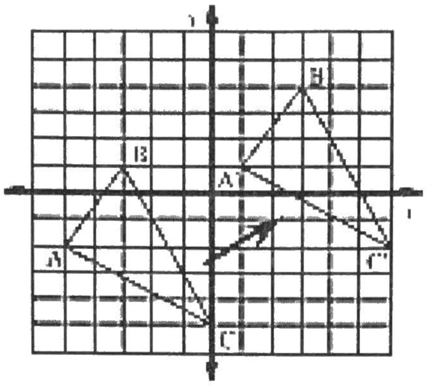
Day 1 – Transformations

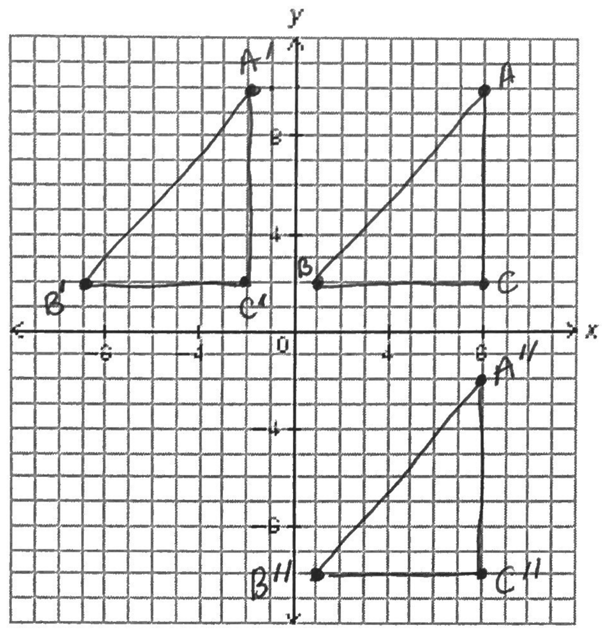
There are many different ways to move a figure on the coordinate plane. Some movements keep the figure the same size and some may make the figure bigger or smaller. These "movements" are called transformations. **Transformations** are the mapping or movement of all the points in a figure on the coordinate plane.



When a figure is the original figure, it is called the **pre-image**. The prefix "pre" means Before. In the above picture, we would label the points as A, B, and C.

When a figure has been transformed, it is called the **image**. We would label the new points as A', B', and C'. We would say that points A, B, and C have been mapped to the new points A', B', and C'

Exploring Translations



A. Graph triangle ABC by plotting points A(8, 10), B(1, 2), and C(8, 2).

B. Translate triangle ABC 10 units to the left to form triangle A'B'C' and write new coordinates.

C. Translate triangle ABC 12 units down to form triangle A''B''C'' and write new coordinates.

Coordinates of Triangle ABC	Coordinates of Triangle A'B'C'	Coordinates of Triangle A"B"C"
A (8, 10)	(8-10, 10) (-2, 10)	(8, 10-12) (8, -2)
B (1, 2)	(1-10, 2) (-9, 2)	(1, 2-12) (1, -10)
C(8, 2))	(8-10, 2) (-2, 2)	(8, 2-12) (8, -10)

↔ changes x ↕ changes y

Observation: Did the figures change size or shape after each transformation?

no change in size or shape.

You observed that your four triangles maintained the same shape and size. When a figure keeps the same size and shape, it is called a **rigid transformation.**

With your experiment, you were performing a translation. A **translation** is a slide that maps all points of a figure the same distance in the same direction. A translation can slide a figure horizontally, vertically, or both.

Rule for Translations: (x, y) → (x + a, y + b)

a → left or right translations (horizontally)
b → up or down translations (vertically)

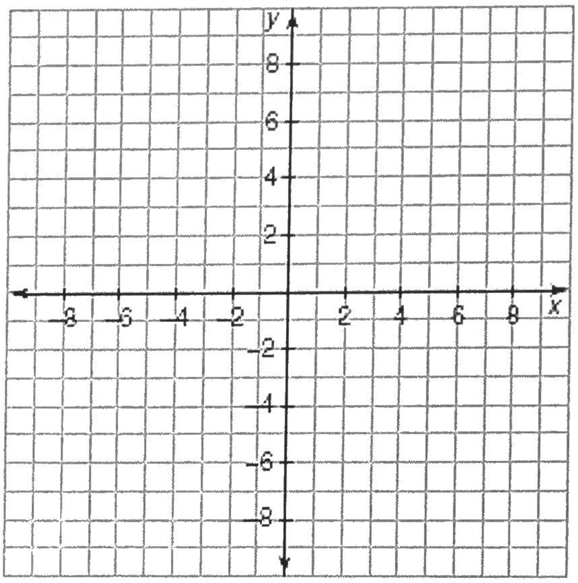
Practice with Translations

Practice:

a. ΔABC has vertices A(1, 2), B(3, 6), and C(9, 7). What are the vertices after the triangle is translated 4 units left?

Rule: (x − 4, y)

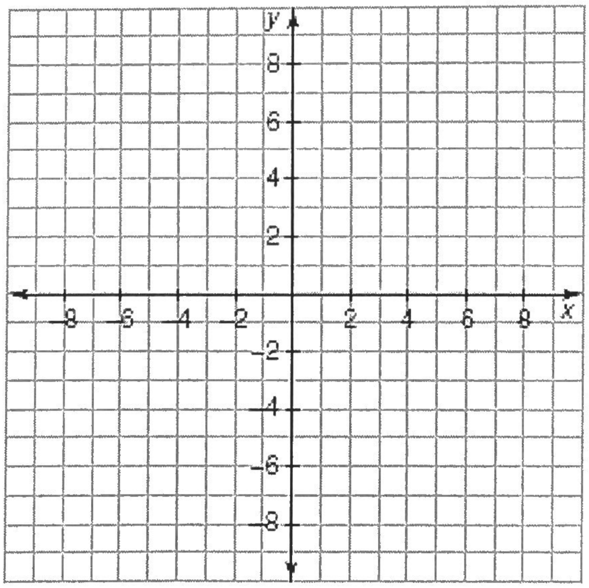
New A′(−3, 2) New B′(−1, 6) New C′(5, 7)



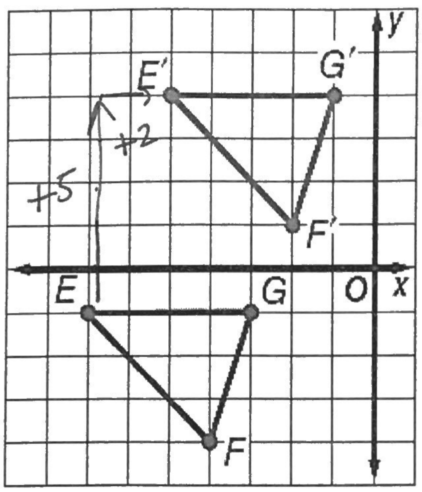
b. ΔXYZ has vertices X(-5, 1), Y(-7, -4), and Z(-2, -4). What are the vertices after the triangle is translated 1 unit right and 6 units up?

Rule: (x + 1, y + 6)

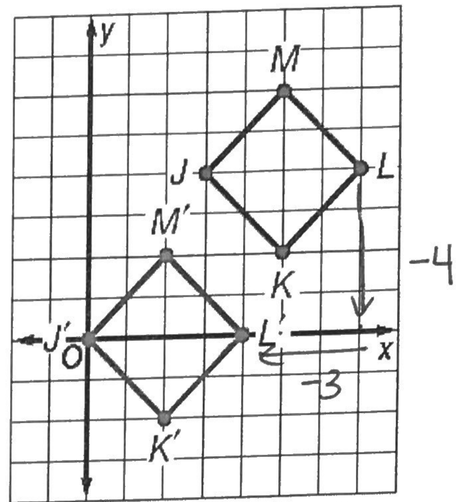
~~New Points~~: Points: X′(−4, 7) Y′(−6, 2) Z′(−1, 2)



c. Name the rule for the given figures:

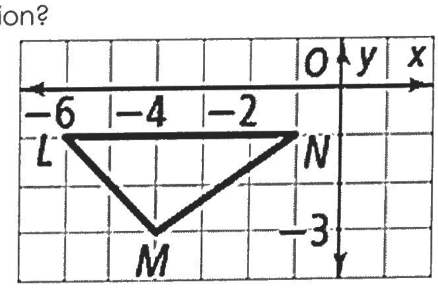


$(x, y) \rightarrow (x+2, y+5)$



$(x, y) \rightarrow (x-3, y-4)$

d. The pre-image of ΔLMN is shown below. The image of ΔLMN is $\Delta L'M'N'$ with L'(1, -2), M'(3, -4), and N'(6, -2). What is a rule that describes the translation?



L(-6, -1) L'(1, -2)

M(-4, -3) M'(3, -4)

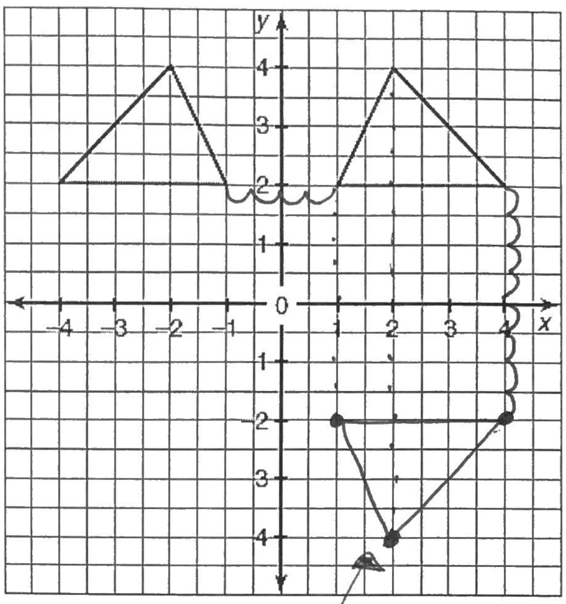
N(-1, -3) N'(6, -2)

$(x, y) \rightarrow (x+7, y-1)$

Reflections

1. Look at the two triangles in the figure. . Do you think they are congruent?

yes - same size
same shape



Figures that are mirror images of each other are called reflections. A **reflection** is a transformation that "flips" a figure over a reflection line. A **reflection line** is a line that acts as a mirror so that corresponding points are the same distance from the mirror. Reflections maintain shape and size; they are our second type of rigid transformation.

⊥ to the line of reflect.

2. What do you think the reflection line is in the diagram?

y-axis / x=0

3. Draw a triangle that would be a reflection over the x-axis.

4. What do you notice about the reflected triangles' points in relation to the pre-image?

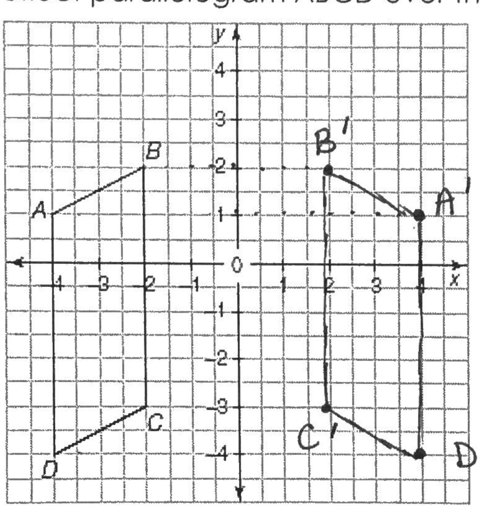
over the y-axis (the x-values became opposites)

over the x-axis (the y-values became opposites)

Reflection over y-axis

Change x

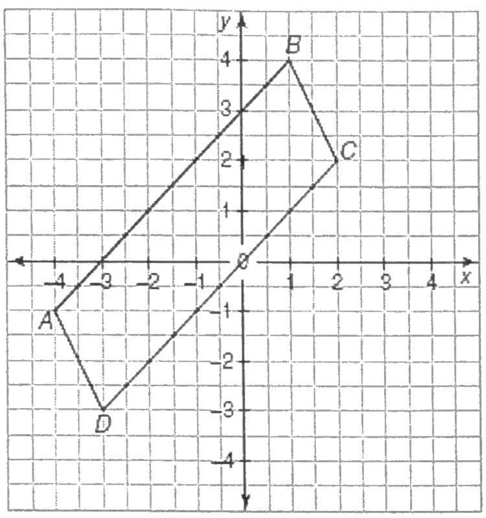
Reflect parallelogram ABCD over the y-axis using reflection lines. Record the points in the table.



	Pre-Image	Image
A	(-4, 1)	(4, 1)
B	(-2, 2)	(2, 2)
C	(-2, -3)	(2, -3)
D	(-4, -4)	(4, -4)
Rule	(x, y)	(-x, y)

Reflection over x-axis Change y

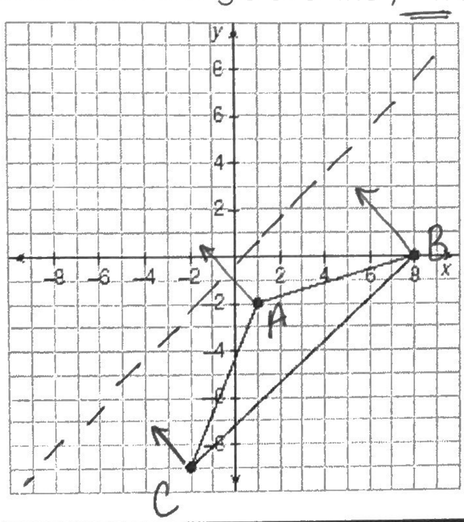
Reflect parallelogram ABCD over the x-axis using reflection lines. Record the points in the table



	Pre-Image	Image
A	(-4, -2)	(-4, 2)
B	(1, 4)	(1, -4)
C	(2, 2)	(2, -2)
D	(-3, -3)	(-3, 3)
Rule	(x, y)	(x, -y)

Reflection over y = x (switch x, y)

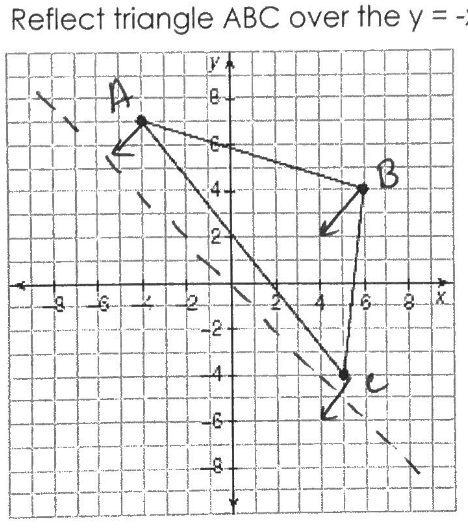
Reflect the triangle over the y = x using reflection lines. Record the points in the table



	Pre-Image	Image
A	(1, -2)	(-2, 1)
B	(8, 0)	(0, 8)
C	(-2, -9)	(-9, -2)
Rule	(x, y)	(y, x)

Reflection over y = -x (Switch + opposites)

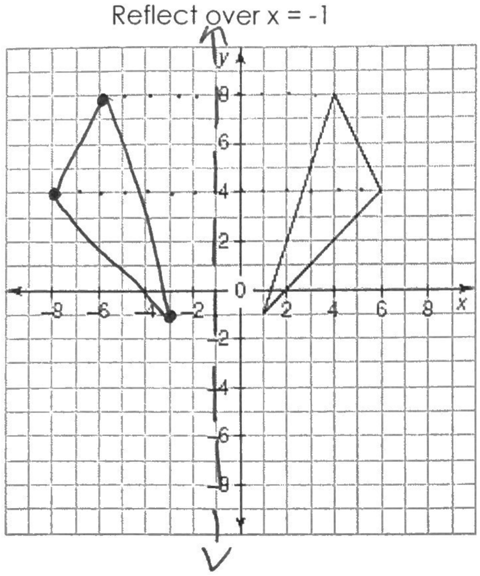
Reflect triangle ABC over the y = -x using reflection lines. Record the points in the table



	Pre-Image	Image
A	(-4, 7)	(-7, 4)
B	(6, 4)	(-4, -6)
C	(5, -4)	(4, -5)
Rule	(x, y)	(-y, -x)

Reflection over Horizontal and Vertical Lines (Count distance)

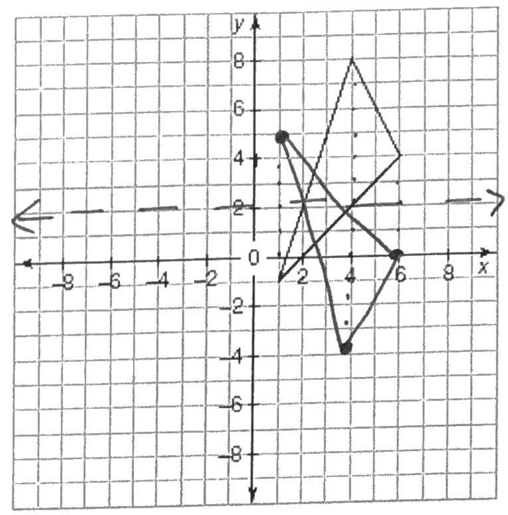
Reflect over x = -1



X = -1

x	y
-1	-1
-1	0
-1	1

Reflect over y = 2



y = 2

x	y
-1	2
0	2
1	2

Practice with Reflections

Given triangle MNP with vertices of M(1, 2), N(1, 4), and P(3, 3), reflect across the following lines of reflection:

x- axis (x, y)→	y-axis (x, y)→	y = x (x, y)→	y = -x (x, y)→
M(1, 2) → (1, -2)	M(1, 2) → (-1, 2)	M(1, 2) → (2, 1)	M(1, 2) → (-2, -1)
N(1, 4)→ (1, -4)	N(1, 4)→ (-1, 4)	N(1, 4)→ (4, 1)	N(1, 4)→ (-4, -1)
P(3, 3)→ (3, -3)	P(3, 3)→ (-3, 3)	P(3, 3)→ (3, 3)	P(3, 3)→ (-3, -3)