Unit 1: Basics of Geometry and Proofs

After completion of this unit, you will be able to…

Learning Target #1: Recognize and Use Geometric Segment and Angle Relationships

- Name angles and lines
- Define and recognize the following relationships
    - Complementary and Supplementary Angles
    - Linear Pair
    - Vertical Angles
    - Midpoint
    - Angle and Segment Bisector
    - Angle Addition and Segment Addition
    - Perpendicular Lines
    - Parallel Line Relationships (Alt. Int, Alt. Ext, Consecutive Int., Consecutive Ext., Corresponding)
- Use the relationships to find missing segment lengths and angles

Learning Target #2: Algebraic and Geometric Proof

- Prove algebraically a geometric relationship using a two column proof
- Prove theorems about lines and angles using a two column proof
- Prove theorems about parallel lines using a two column proof

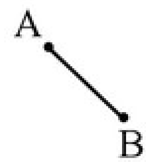
Basics of Geometry

Naming Angles and Lines

Point

Points are named with capital letters.

Line Segment



Two points are connected with a straight line. This line segment can be named $\overline{AB}$ or $\overline{BA}$.

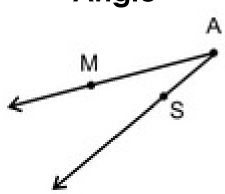
Line

A line does not have a beginning or end point. Lines are named using two points on the line. This line can be named $\overleftrightarrow{VW}$ or $\overleftrightarrow{WV}$.

Ray

Rays start with a point but continue to infinity in one direction. Rays are named using its starting point and one other point on the ray. The ray can be named $\overrightarrow{AB}$ but NOT $\overrightarrow{BA}$.

Angle



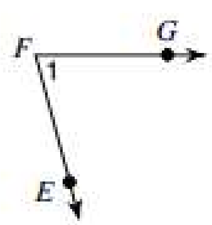
Angles are made up of two rays that have the same beginning point. The point is called the vertex and the two rays are called the side of the angle. Angles can be name in ways:

One Letter (if the vertex is not shared): $\angle A$

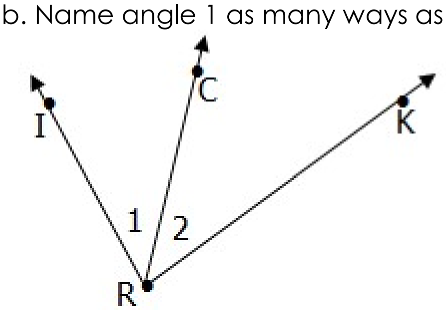
Number (if given): $\angle 1$

Three Letters (vertex is middle letter): $\angle MAS$ or $\angle SAM$

a. Name the angle in four ways:



b. Name angle 1 as many ways as possible:



TYPES OF ANGLES

Acute Angles
Acute angles have measures between ____ & ____

Obtuse Angles
Obtuse Angles have measures between ____ & ____

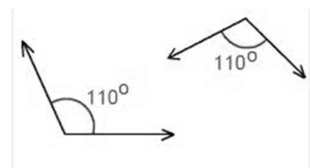
Right Angles
Right Angles measure exactly ____

Straight Angles
Straight Angles measure exactly ____

Important Geometry Symbols

$\angle$ Angle

Δ Triangle



Congruent Angles

$\cong$ Congruent (same shape & size)

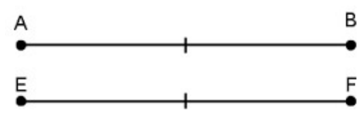
° Degrees

$\perp$ Perpendicular (90 degrees)

m Measure of

|| Parallel

~ Similar



Congruent Segments

Using Geometry Terminology

A **conditional statement** (if-then) is a statement that contains a hypothesis (if) and conclusion (then).
Ex. **If** a student plays basketball, **then** they are an athlete.

A **converse** is a statement that has the hypothesis and conclusion switched around.
Ex. **If** a student is an athlete, **then** they play basketball. (Is this true?)

A **postulate** is a statement that is accepted as true without proof.

A **theorem** is a statement that must be proven before it can be accepted as true. We are going to prove many theorems throughout this unit. We will prove a few of the following relationships on Day 3.

Practice: Take the following statement: *I do my homework; I get my allowance*, and write it in if-then form and then write the converse of it.

Supplementary and Complementary Angles

Complementary Angles: Two or more angles whose sum of measures equals 90°.

40° and 50° angles are complementary angles because 40° + 50° = 90°.

Example: A 30° angle is called the complement of the 60° angle.
Similarly, the 60° angle is the complement of the 30° angle.

Practice: Find the **complement** of each angle.

a. 35°

b) Two angles, 2x° and 3x° are complementary. Find the value of x and each angle.

Supplementary Angles: Two or more angles whose sum of measures equals 180°.

60° and 120° angles are supplementary angles because 60° + 120° = 180°.

Example: A 70° angle is called the supplement of the 110° angle.
Similarly, the 110° angle is the supplement of the 70° angle.

Practice: Find the **supplement** of each angle.

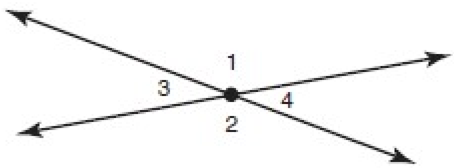
a.) 126°

b) Two angles, 4x° and 6x° are supplementary. Find the value of x and each angle.

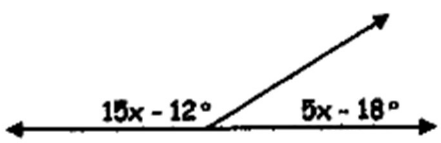
Special Pairs of Angles

<u>Linear Pair:</u> Two adjacent (next to) angles whose noncommon sides are opposite rays. A linear pair also forms a line (supplementary).

a. Name all the linear pairs in the diagram below:

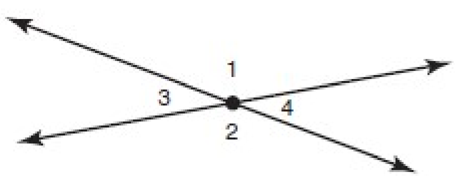


b. Solve for x:



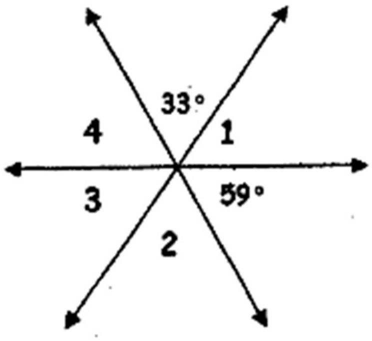
<u>Vertical Angles:</u> Two nonadjacent angles that are formed by two intersecting lines. Vertical angles are congruent.

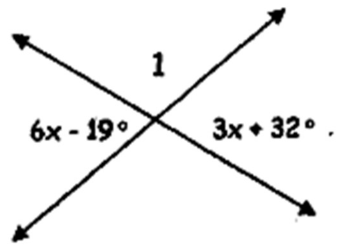
a. Name all the vertical angles in the diagram below:



b. Find the measure of angles 1, 2, 3, and 4.

c. Solve for x. Then determine the measure of angle 1.

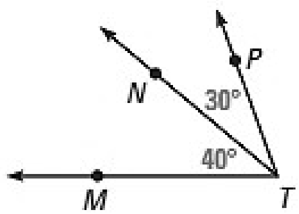




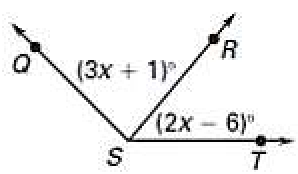
Angle & Segment Relationships

Angle Addition Postulate: If point D lies in the interior of $\angle$ ABC, then m$\angle$ ABD + m$\angle$ DBC = m$\angle$ ABC.

a. Find the measure of $\angle$ PTM:

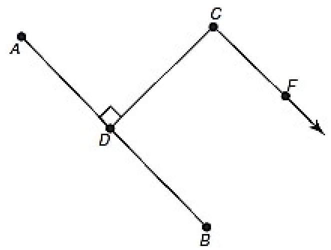


b. Given $m\angle QST = 135°$, find $m\angle QSR$.

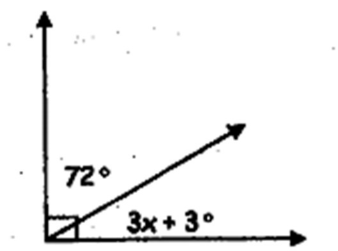


Perpendicular: Two lines, rays, or segments that intersect to form a 90° angle.

a. Name all the angles you know are right angles.

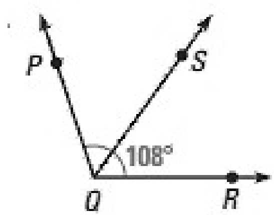


b. Solve for x.

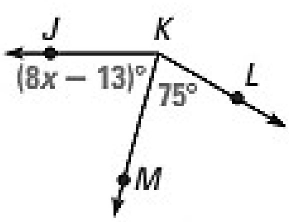


<u>Angle Bisector</u>: A ray that divides an angle into two congruent angles (two angles with equal measure).

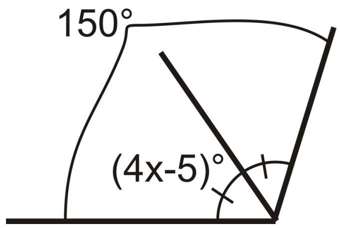
a. $\overrightarrow{QS}$ bisects $\angle PQR$. Find $m\angle PQS$.



b. $\overrightarrow{KM}$ bisects $\angle JKL$. Find the value of x.



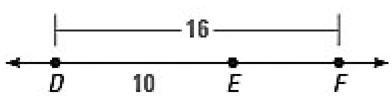
c. Solve for x.



Segment Relationships

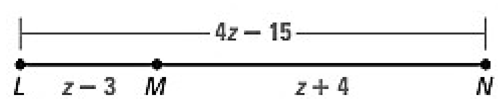
Segment Addition Postulate: If point B is on $\overline{AC}$, and between points A and C, then $\overline{AB} + \overline{BC} = \overline{AC}$.

a. Use the diagram to find $\overline{EF}$.



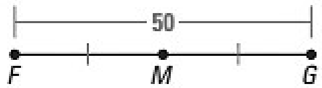
b. Write an expression for AC.

c. Find the value of z.

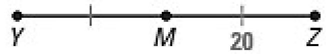


Midpoint: Point that divides the segment into two congruent segments.

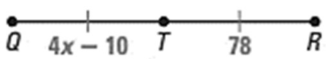
a. Find $\overline{FM}$ and $\overline{MG}$.



b. Find $\overline{YM}$ and $\overline{YZ}$.

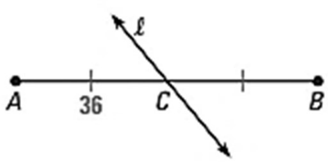


c. T is the midpoint of $\overline{QR}$. Solve for x.

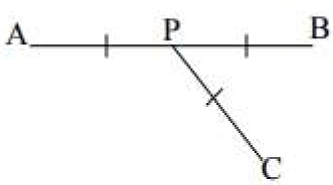


<u>Segment Bisector:</u> A line, line segment, or ray that divides the line segment into two line segments of equal length.

a. Find $\overline{CB}$ and $\overline{AB}$.

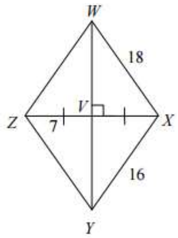


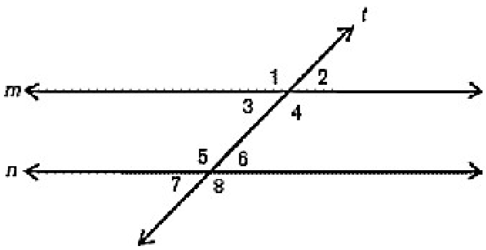
b. Determine if you have enough information to determine if $\overline{PC}$ is the segment bisector of $\overline{AB}$. Explain why or why not.



<u>Perpendicular Bisector:</u> A line, line segment, or ray that intersects at the midpoint of a line segment at a 90 degree angle.

a. Determine if you have enough information to determine if $\overline{WY}$ is the perpendicular bisector of $\overline{ZX}$. Explain why or why not.





Parallel Lines

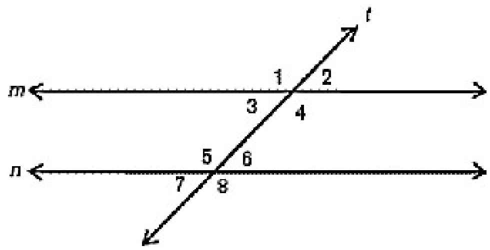
Definition:
Parallel Lines are two lines that never intersect. They are always the same distance apart.

Alternate Exterior Angles

Definition:

Two angles in the __________________ of the parallel lines and on __________________ sides.

Alternate Exterior Angles Theorem:
If 2 ______________ ______________ are cut by a transversal, then the pairs of alternate exterior angles are ____________.



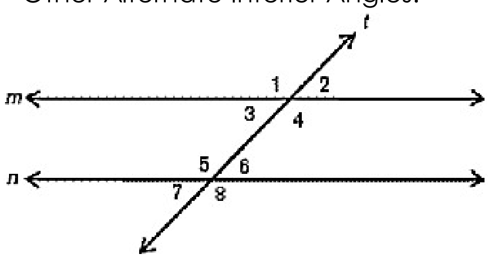
Other Alternate Exterior Angles:

Alternate Interior Angles

Definition:
Two angles in the __________________ of the parallel lines and on __________________ sides.

Alternate Interior Angles Theorem:
If 2 ______________ ______________ are cut by a transversal, then the pairs of alternate interior angles are ____________.

Other Alternate Interior Angles:



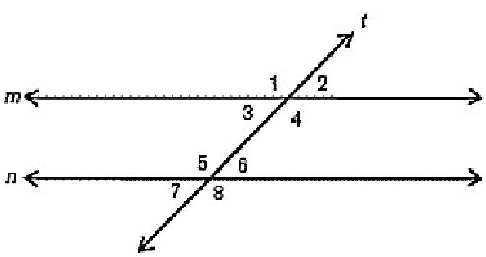
Consecutive (Same Side) Exterior Angles

Definition:
Two angles in the __________________ of the parallel lines and on __________________ sides.

Consecutive (Same Side) Exterior Angles Theorem:
If 2 ______________ ______________ are cut by a transversal, then the pairs of consecutive exterior angles are _______________.

Other Same Side Exterior Angles:

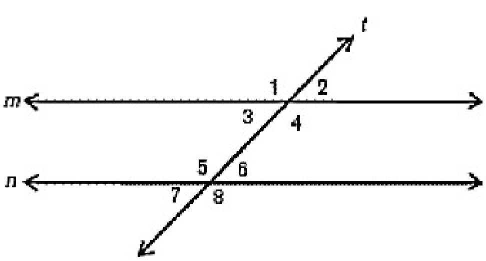
Consecutive (Same Side) Interior Angles



Definition:
Two angles in the __________________ of the parallel lines and on __________________ sides.

Consecutive (Same Side) Interior Angles Theorem:
If 2 ______________ ______________ are cut by a transversal, then the pairs of consecutive interior angles are _______________.

Other Same Side Interior Angles:

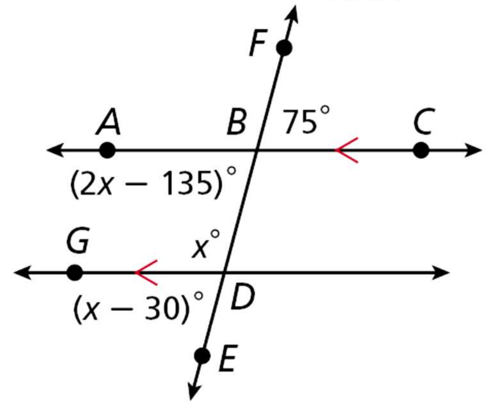


Corresponding Angles

Definition:
Two angles that lie in the________________________________.

Corresponding Angles Postulate:

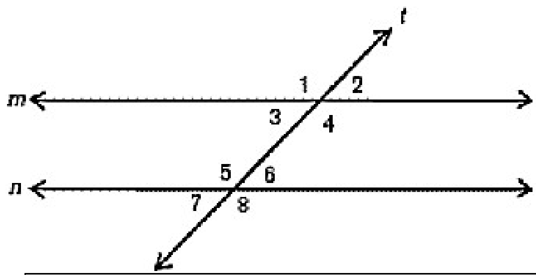
If 2 ______________________________ are cut by a transversal, then the pairs of corresponding angles are ______________.



Other Corresponding Angles:

Transversal

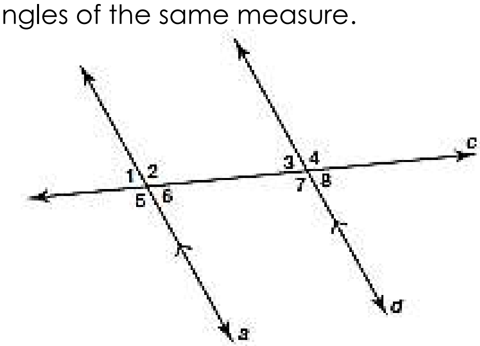
Definition:
A ________________ is a line that intersects two or more coplanar lines at different points.



Summary of Parallel Line Relationships

Relationships with Parallel & Non Parallel Lines		
Angle Type	*Parallel Lines*	*Non Parallel Lines*
Alternate Exterior Angles		
Alternate Interior Angles		
Same Side Exterior Angles		
Same Side Interior Angles		
Corresponding Angles		
Vertical Angles		

Practice:
1. If the measure of angle 1 = 67° and a is parallel to *d,* find all other angles of the same measure.

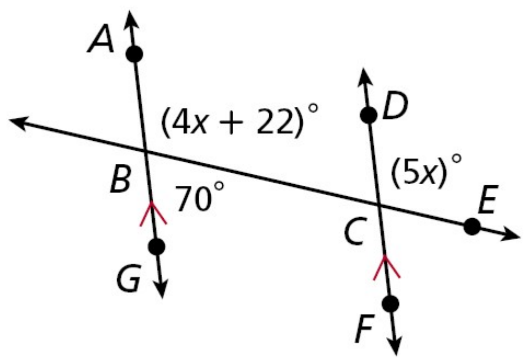


2. Find the measure of the following:

a. Solve for x: b. $m\angle ECF$ c. $m\angle DCE$

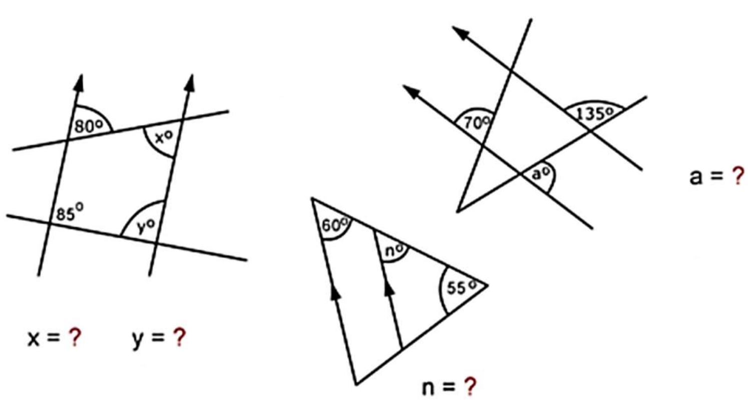
3. Find the measure of the following:

a. Solve for x: b. $m\angle EDG$



c. How did you solve for x? What is another way you could have solved for x?

4. What is the value of x, y, n, and a?



x = ______________

y = ______________

n = ______________

a = ______________

Intro to Proofs (Algebraic)

1. Solve the following equation.
   Justify each step as you solve it.

$2(4x - 3) - 8 = 4 + 2x$	

2. Rewrite your proof so it is "formal" proof.

$2(4x - 3) - 8 = 4 + 2x$	

When writing an algebraic proof, you create a chain of logical steps that move from the hypothesis to the conclusion of the conjecture you are proving. By proving the conclusion is true, you have proven the original conjecture is true.

When writing a proof, it is important to justify each logical step with a reason. You can use symbols and abbreviations, but they must be clear enough so that anyone who reads your proof will understand them.

Two Column Proofs

-
-
-

Practice #1:

GIVEN ▶ $\overline{AB} \cong \overline{BC}$, $\overline{CD} \cong \overline{BC}$

PROVE: x = 6

What Is the length of $\overline{AB}$?

STATEMENTS	REASONS

What Is the length of $\overline{CD}$?

Practice #2:

GIVEN ▶ $\overline{ST} \cong \overline{SR}, \overline{QR} \cong \overline{SR}$

PROVE: x = 1

STATEMENTS	REASONS

Practice: #3

. **GIVEN** ▶ $PR = 46$

Prove: x = 7

STATEMENTS	REASONS

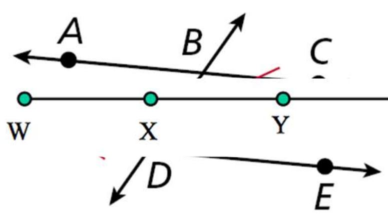
Practice: #4

GIVEN: ∠ABD and ∠BDE are alternate interior angles.

Prove: $m\angle DBC = 120°$

STATEMENTS	REASONS

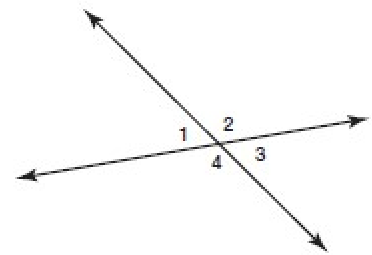
Practice: #5



GIVEN: $\overline{WX} = \overline{YZ}$
Y is the midpoint of $\overline{XZ}$.

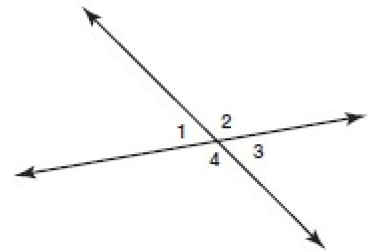
Prove: $\overline{WX} = \overline{XY}$

STATEMENTS	REASONS



Geometric Proofs

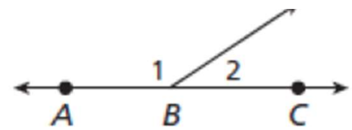
When writing a geometric proof, you use deductive reasoning to create a chain of logical steps that move from the hypothesis to the conclusion of the conjecture you are proving. By proving the conclusion is true, you have proven the original conjecture is true.



When writing a proof, it is important to justify each logical step with a reason. You can use symbols and abbreviations, but they must be clear enough so that anyone who reads your proof will understand them.

Practice:
Fill in the blanks to complete a two column proof of the Linear Pair Theorem.

Given: Angle 1 and 2 form a linear pair.
Prove: Angle 1 and 2 are supplementary.

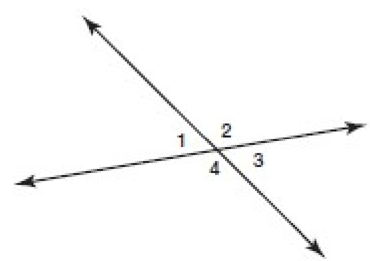


Statements	**Reasons**
1. ∠1 and ∠2 form a linear pair.	1. Given
2. BA and BC form a line.	2. ______________________
3. m∠ABC = 180°	3. ______________________
4. ______________________	4. Angle Addition Postulate
5. ______________________	5. Substitution
6. ∠1 and ∠2 are supplementary	6. ______________________

Prove the Vertical Angles Theorem using a two column proof.

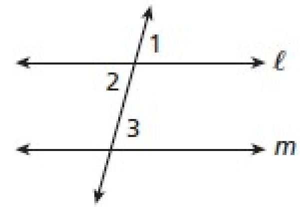
Given: Angle 4 and 1 are a linear pair.
Angle 1 and 2 are a linear pair.
Prove: ∠2 ≅ ∠4

Statements	**Reasons**
1. ______________________	1. Given
2. ______________________	2. Given
3. ______________________	3. Linear Pair Theorem
4. ______________________	4. Linear Pair Theorem
5. ______________________ Supplementary Angles	5. Definition of
6. ______________________ Supplementary Angles	6. Definition of
7. ______________________	7. Substitution
8. ______________________	8. Subtraction Property
9. ______________________	9. Definition of Congruent Angles



Prove the Alternate Interior Angles are Congruent Theorem using a two column proof:

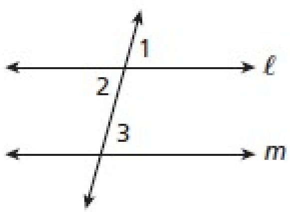
Given: l // m
Prove: ∠ 2 ≅ ∠ 3



Statements	Reasons
1. ________________	1. Given
2. ________________	2. Vertical Angles are Congruent
3. ________________	3. Corresponding Angles Postulate
4. ________________	4. Transitive Property

Prove that Corresponding Angles are congruent using a two column proof:

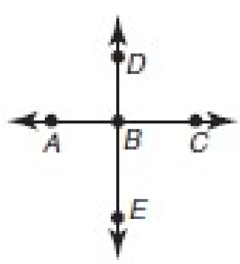
Given: l // m
Prove: ∠ 1 ≅ ∠ 3



Statements	Reasons
1. l // m	1. ________________
2. ∠ 1 ≅ ∠ 2	2. ________________
3. ∠ 3 ≅ ∠ 2	3. ________________
4. ∠ 1 ≅ ∠ 3	4. ________________

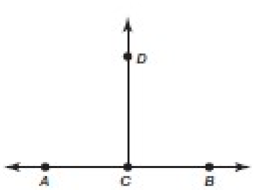
Prove the following using a two column proof:

Given: $\overleftrightarrow{AC} \perp \overleftrightarrow{DE}$
Prove: ∠ABD ≅ ∠CBD



Statements	Reasons
1. ________________	1. Given
2. ________________	2. Definition of Perpendicular Lines
3. ________________	3. Definition of Perpendicular Lines
4. ________________	4. Right ∠s are ≅ OR Transitive Prop.
5. ________________	5. Definition of Congruent Angles

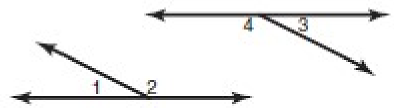
Prove the <u>Right Angle Congruence Theorem</u> using a two column proof.
Given: ∠ACD and ∠BCD are right angles
Prove: ∠ACD ≅ ∠BCD



Statements	Reasons
1. ________________	1. Given
2. ________________	2. ________________
3. m∠ACD = 90°	3. ________________
4. ________________	4. ________________
5. ________________	5. Transitive Property
6. ∠ACD ≅ ∠BCD	6. ________________

Prove the Congruent Supplement Theorem using a two column proof:

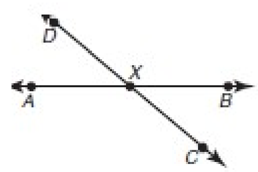
Given: $\angle 2 \cong \angle 4$
Angle 1 is supplementary to angle 2
Angle 3 is supplementary to angle 4
Prove: $\angle 1 \cong \angle 3$



Statements	Reasons
1. $\angle 2 \cong \angle 4$	1. Given
2. ______________	2. Definition of congruent angles
3. ______________	3. Given
4. ______________	4. ______________
5. ______________	5. Defintion of supplementary angles
6. $m\angle 3 + m\angle 4 = 180°$	6. ______________
7. ______________	7. Substitution Property
8. $m\angle 1 + m\angle 2 = m\angle 3 + m\angle 2$	8. ______________
9. ______________	9. Subtraction Property of Equality
10. ______________	10. ______________

Prove the following using a two column proof:

Given: $m\overline{AX} = m\overline{CX}$, $m\overline{BX} = m\overline{DX}$
Prove: $m\overline{AB} = m\overline{CD}$



Statements	Reasons
1. $m\overline{BX} = m\overline{DX}$	1. ______________
2. $m\overline{AX} = m\overline{CX}$	2. ______________
3. $m\overline{AX} + m\overline{BX} = m\overline{CX} + m\overline{BX}$	3. ______________
4. $m\overline{AX} + m\overline{BX} = m\overline{CX} + m\overline{DX}$	4. ______________
5. $m\overline{AX} + m\overline{BX} = m\overline{AB}$	5. ______________
6. $m\overline{CX} + m\overline{DX} = m\overline{CD}$	6. ______________
7. $m\overline{AB} = m\overline{CD}$	7. ______________