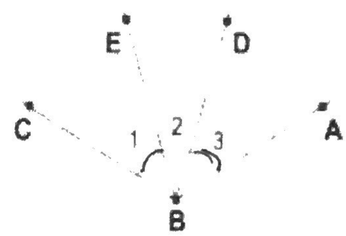
Prove Statements about Segments and Angles

Writing a two-column proof is a formal way of organizing your reasons to show a statement is true. Each reason in the right-hand column is the explanation for the corresponding statement.

Write a two-column proof for the situations below.

Example 1: Given: $m\angle 1 = m\angle 3$

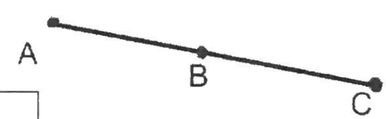
Prove: $m\angle EBA = m\angle DBC$



Statements	Reasons
$m\angle 1 = m\angle 3$	Given
$m\angle EBA = m\angle 3 + m\angle 2$	Angle Addition Postulate
$m\angle EBA = m\angle 1 + m\angle 2$	Substitution
$m\angle 1 + m\angle 2 = m\angle DBC$	Angle Addition Postulate
$m\angle EBA = m\angle DBC$	Transitive Property of Equality

Example 2: Given: AC = AB + AB

Prove: AB = BC



Statements	Reasons
AC = AB + AB	Given
AB + BC = AC	Segment Addition Post.
AB + AB = AB + BC	Transitive Prop.
AB = BC	Subtraction Prop

The reasons used in a proof can include definitions, properties, postulates, and theorems. A Theorem is a statement that can be proven. Once you have proven a theorem, you can use the theorem as a reason in other proofs.

Theorems:

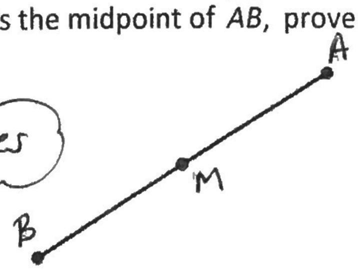
- **Congruence of Segments**
  Segment congruence is reflexive, symmetric, and transitive.
  - **Reflexive** – For any segment AB, $\overline{AB} \cong \overline{AB}$
  - **Symmetric** – If $\overline{AB} \cong \overline{CD}$, then $\overline{CD} \cong \overline{AB}$
  - **Transitive** – If $\overline{AB} \cong \overline{CD}$ and $\overline{CD} \cong \overline{EF}$, then $\overline{AB} \cong \overline{EF}$
- **Congruence of Angles**
  Angle congruence is reflexive, symmetric, and transitive.
  - **Reflexive** – For any angle A, $\angle A \cong \angle A$
  - **Symmetric** – If $\angle A \cong \angle B$ then $\angle B \cong \angle A$
  - **Transitive** – If $\angle A \cong \angle B$ and $\angle B \cong \angle C$, then $\angle A \cong \angle C$

Reminder:

Definition of Congruence

If $\overline{AB} \cong \overline{CD}$, then $m\overline{AB} = m\overline{CD}$.

Example 3: Prove this property of midpoints. If you know that M is the midpoint of $\overline{AB}$, prove that AB is two times AM and AM is one half of AB.



Given: M is the midpoint of $\overline{AB}$

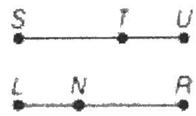
then 2 equal pieces are created

Prove: $AB = 2 \bullet AM$ and $AM = \frac{1}{2}AB$

Statements	Reasons
M is the midpoint of $\overline{AB}$	Given
$\overline{AM} \cong \overline{MB}$	Definition of Midpoint
AM = MB	Definition of congruent segments
AM + MB = AB	Segment Addition Postulate
AM + AM = AB	Substitution
$2 \bullet AM = AB$	Combine like terms
$AM = \frac{1}{2}AB$	Division Property

$\underbrace{1AM + 1AM}_{2AM}$

Example 4: Complete the proof below.



Given: $\overline{SU} \cong \overline{LR}$, $\overline{TU} \cong \overline{LN}$

Prove: $\overline{ST} \cong \overline{NR}$

Statements	Reasons
$\overline{SU} \cong \overline{LR}$, $\overline{TU} \cong \overline{LN}$	Given
SU = LR, TU = LN	Definition of Congruent Segments
SU = ST + TU	Segment Addition Postulate
LR = LN + NR	Segment Addition Postulate
ST + TU = LN + NR	Transitive Prop.
ST + LN = LN + NR	~~Subtraction Prop.~~ Substitution
ST = NR	Subtraction Prop
$\overline{ST} \cong \overline{NR}$	Def. of Congruent Segments

They share SU = LR

Skills Practice

Name the property illustrated by the statement.

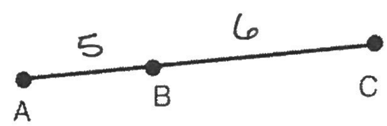
1. If $\overline{DG} \cong \overline{CT}$, then $\overline{CT} \cong \overline{DG}$ Property: Symmetric Prop.
2. $\angle VWX \cong \angle VWX$ Property: Reflexive Prop.
3. If $\overline{JK} \cong \overline{MN}$ and $\overline{MN} \cong \overline{XY}$, then $\overline{JK} \cong \overline{XY}$ Property: Transitive Prop
4. YZ = ZY Property: Symmetric Prop.

Use the property complete the statement.

5. Reflexive Property of Congruence: $\overline{SE}$ $\cong \overline{SE}$
6. Symmetric Property of Congruence: If $\angle JKL \cong \angle RST$, then $\angle RST \cong \angle JKL$
7. Transitive Property of Congruence: If $\angle F \cong \angle J$ and $\angle J \cong \angle J$, then $\angle F \cong \angle L$

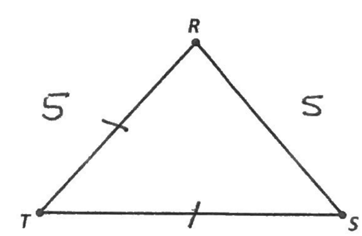
Complete the proofs below.

8. Given: AB = 5, BC = 6
   Prove: AC = 11



Statements	Reasons
AB = 5, BC = 6	Given
AB + BC = AC	Segment Addition Postulate
5 + 6 = AC	Substitution Property
11=AC	Combine Like terms / Simplify
AC=11	Symmetric Prop.

9. Given: RT = 5, RS = 5, $\overline{RT} \cong \overline{TS}$
   Prove: $\overline{RS} \cong \overline{TS}$



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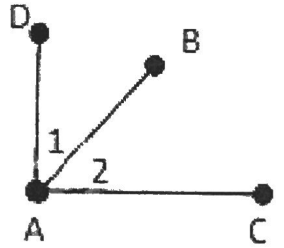
Statements	Reasons
RT = 5, RS = 5, $\overline{RT} \cong \overline{TS}$	Given
RT = RS	Transitive Property of Equality
RT = TS	Def. of Congruent Segments
RS = TS	Substitution
$\overline{RS} \cong \overline{TS}$	Def of Congruent Segments

10. Given: $m\angle 1 = 45°$ and $m\angle 2 = 45°$

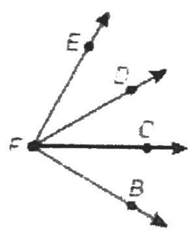
Prove: $\overline{AB}$ is the bisector of $\angle DAC$



Statements	Reasons
$m\angle 1 = 45°$ and $m\angle 2 = 45°$	Given
$m\angle 1 = m\angle 2$	~~Substitution~~ Property of Equality Transitive
$\angle 1 \cong \angle 2$	Def. of Congruent ∠'s
$\overline{AB}$ is the bisector of $\angle DAC$	Def. of Angle Bisector

11. Given: $\overline{FD}$ bisects $\angle EFC$ and $\overline{FC}$ bisects $\angle DFB$

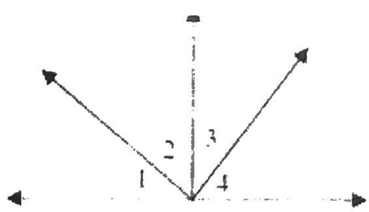
Prove: $\angle EFD \cong \angle CFB$



Statements	Reasons
$\overline{FD}$ bisects $\angle EFC$ ~~FC~~	Given
$\overline{FC}$ bisects $\angle DFB$	Given
$\angle EFD \cong \angle DFC$	Def. of Angle Bisector
$\angle DFC \cong \angle CFB$	Def. of Angle Bisector
$\angle EFD \cong \angle CFB$	Transitive Property of Congruence

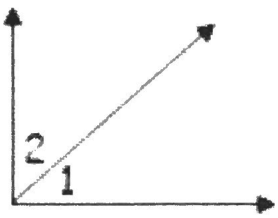
12. Given: $\angle 1$ *and* $\angle 2$ are complementary, $\angle 1 \cong \angle 3$, and $\angle 2 \cong \angle 4$

Prove: $\angle 3$ *and* $\angle 4$ are complementary



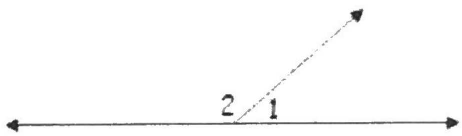
Statements	Reasons
$\angle 1$ and $\angle 2$ are Complementary	Given
$\angle 1 \cong \angle 3$	Given
$\angle 2 \cong \angle 4$	Given
$\angle 1 = \angle 3$	Def. of Congruence
$\angle 2 = \angle 4$	Def. of Congruence
$m\angle 1 + m\angle 2 = 90$	Def. of Complementary
$m\angle 3 + m\angle 4 = 90$	Substitution Property
$\angle 3$ and $\angle 4$ are Complementary	Def. of Complementary Angles

13. Given: $\angle 1$ *and* $\angle 2$ are complementary and $m\angle 2 = 46°$
Prove: $m\angle 1 = 44°$



Statements	Reasons
$\angle 1$ and $\angle 2$ are complementary	Given
$m\angle 2 = 46$	Given
$m\angle 1 + m\angle 2 = 90$	Def. of Complementary Angles
$m\angle 1 + 46 = 90$	Substitution Prop
$m\angle 1 = 44°$	Subtraction Prop.

14. Given: $m\angle 1 + m\angle 2 = 180°$ and $m\angle 1 = 62°$
Prove: $m\angle 2 = 118°$

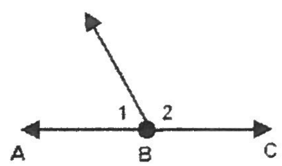


Statements	Reasons
$m\angle 1 + m\angle 2 = 180$	Given
$m\angle 1 = 62$	Given
$62 + m\angle 2 = 180$	Substitution Prop.
$m\angle 2 = 118$	Subtraction Prop.

Prove Angle Pair Relationships

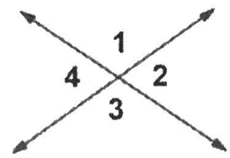
When two lines intersect, pairs of vertical angles and linear pairs are formed.

Linear Pair Postulate: If two angles form a linear pair, then they are supplementary.



$\angle 1$ and $\angle 2$ form a linear pair, so $\angle 1$ and $\angle 2$ are supplementary and $m\angle 1 + m\angle 2 = 180°$

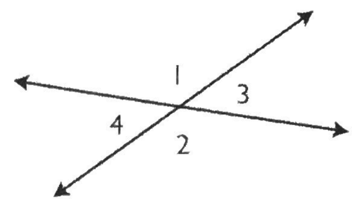
Vertical Angles Congruence Theorem: Vertical angles are congruent.



SKIP

$\angle 1 \cong \angle 3, \angle 2 \cong \angle 4$

Prove the Vertical Angles of Congruence Theorem-



Given: $\angle 4$ and $\angle 3$ are vertical angles

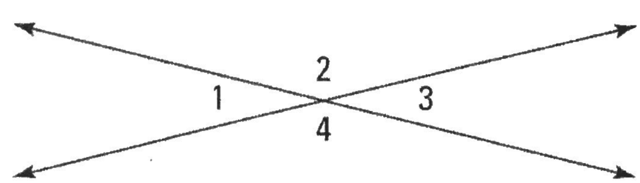
Prove: $\angle 4 \cong \angle 3$

Statements	Reasons
$\angle 4$ and $\angle 3$ are Vertical $\angle$'s	Given
$\angle 2$ and $\angle 4$ are a linear pair	Def. of a Straight line
$\angle 2$ and $\angle 3$ are a linear pair	Def. of Linear Pair
$\angle 2$ and $\angle 4$ are supplementary	Def. of a Linear Pair
$\angle 2$ and $\angle 3$ are Supplementary	Linear Pair Postulate
$m\angle 2 + m\angle 4 = 180$	Def. of Supplementary
$m\angle 2 + m\angle 3 = 180°$	Def. of Supplementary
$m\angle 2 + m\angle 3 = m\angle 2 + m\angle 4$	Substitution Property
$m\angle 3 = m\angle 4$	Subtraction Property
$\angle 3 \cong \angle 4$	Def. of $\cong$ $\angle$'s

$\angle 4 \cong \angle 3$ Symmetric Prop.

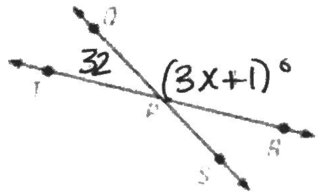
Examples:

Use the diagram below to answer the following questions. Note that the diagram is not drawn to scale.



1. If $m\angle 1 = 112°$, find $m\angle 2, m\angle 3,$ *and* $m\angle 4$.
2. If $m\angle 2 = 67°$, find $m\angle 1, m\angle 3,$ *and* $m\angle 4$.
3. If $m\angle 4 = 71°$, find $m\angle 1, m\angle 2,$ *and* $m\angle 3$.

4. Multiple Choice: Which equation can be used to find x?



A. $32+(3x+1)=90$
B. $32+(3x+1)=180$
C. $32=3x+1$
D. $3x+1=212$

5. Solve for x in Example 4 above.

$32+3x+1=180$
$3x+33=180$
$3x=147$
$x=49$

6. Find $m\angle TPS$ in Example 4 above.

$m\angle TRS \cong m\angle QPS$
$3(49)+1=148°$