

## Algebraic Proofs

Warm Up: Solve for x.

$$1. \begin{array}{r} 3x + 5 = 17 \\ -5 \quad -5 \\ \hline 3x = 12 \end{array} \quad \begin{array}{r} 3x = 12 \\ \div 3 \\ \hline x = 4 \end{array} \quad \boxed{x = 4}$$

$$2. \begin{array}{r} 4x - 5 = 8x + 3 \\ -4x \quad -4x \\ \hline -5 = 4x + 3 \end{array} \quad \begin{array}{r} -5 = 4x + 3 \\ -3 \quad -3 \\ \hline -8 = 4x \\ \frac{-8}{4} = \frac{4x}{4} \end{array} \quad \boxed{x = -2}$$

$$3. \begin{array}{r} 2(x-5) - 20 = 0 \\ 2x - 10 = 20 \\ +10 \quad +10 \\ \hline 2x = 30 \\ \frac{2x}{2} = \frac{30}{2} \end{array} \quad \boxed{x = 15}$$

$$4. \begin{array}{r} \frac{x+8}{5} = -6 \\ 5 \left( \frac{x+8}{5} \right) = -6 \cdot 5 \\ x+8 = -30 \\ -8 \quad -8 \\ \hline x = -38 \end{array} \quad \boxed{x = -38}$$

Steps:

1. Distribute
2. Add 20 to both sides
3. Add 10 to both sides
4. Divide 2 on both sides.

Steps:

1. Mult. both sides by 5
2. subtract 8 from both
3. OH! Dme! (U)

### Proofs

- Justifications are used for EVERY step!
- Here is a list of justifications that CAN be used. There are other justifications that you learned and can use as well.

### Properties of Equality

Addition Property of Equality	If $a = b$ , then $a + c = b + c$ .
Subtraction Property of Equality	If $a = b$ , then $a - c = b - c$ .
Multiplication Property of Equality	If $a = b$ , then $ac = bc$ .
Division Property of Equality	If $a = b$ and $c \neq 0$ , then $\frac{a}{c} = \frac{b}{c}$ .
Reflexive Property of Equality	$a = a$
Symmetric Property of Equality	If $a = b$ , then $b = a$ .
Transitive Property of Equality	If $a = b$ and $b = c$ , then $a = c$ .
Substitution Property of Equality	If $a = b$ , then $b$ can be substituted for $a$ in any expression.

Reminder

- Def. of Vertical Angles
- Def. of Supplementary
- Def. of Complementary
- Addition Postulate
- Seg. Add. Postulate

Distributive Property

$$a(b + c) = ab + ac$$

Combine like Terms/Simplify

$$x + 2 + 3x - 5 = 4x - 3$$

Def. of Congruence

$$\text{If } \overline{AB} \cong \overline{DC}, \text{ then } AB = DC$$

**Example 1:**

Complete a two column proof to prove the following:

Given:  $6x - 3 = 3(x + 1)$

Prove:  $x = 2$

<u>Statement</u>	<u>Reason</u>
1. $6x - 3 = 3(x + 1)$	1. given
2. $6x - 3 = 3x + 3$	2. Distributive Prop.
3. $3x - 3 = 3$	3. Subt. Prop. of Equal.
4. $3x = 6$	4. Add. Prop. of Equality
5. $x = 2$	5. Division Prop. of Equ.

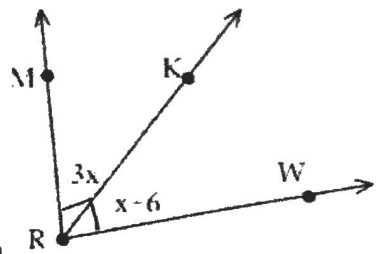
**Example 2:**

Complete a two column proof to prove the following:

Given:  $m\angle MRK = 3x$ ,  $m\angle KRW = x + 6$ ,  $m\angle MRW = 90^\circ$

Prove:  $m\angle MRK = 63^\circ$

Mark figure if not done for you!



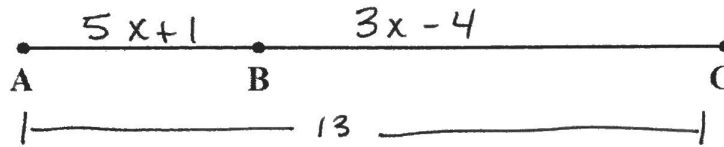
<u>Statement</u>	<u>Reason</u>
1. $m\angle MRK = 3x$ $m\angle KRW = x + 6$ $m\angle MRW = 90$	1. Given
2. $m\angle MRK + m\angle KRW = m\angle MRW$	2. Angle Addition Postulate
3. $3x + x + 6 = 90$	3. Substitution Prop.
4. $4x + 6 = 90$	4. Combine Like Terms / Simplify
5. $4x = 84$	5. Subtraction Prop.
6. $x = 21$	6. Division Prop.
7. $m\angle MRK = 3(21)$	7. Substitution Prop.
8. $m\angle MRK = 63^\circ$	8. Simplify

**Example 3:**

Complete a two column proof to prove the following:

Given:  $AB = 5x + 1$ ,  $BC = 3x - 4$ ,  $AC = 13$

Prove:  $BC = 2$



Statement	Reason
1. $AB = 5x + 1$ $BC = 3x - 4$ $AC = 13$	1. given
2. $AB + BC = AC$	2. Segment Addition Prop.
3. $5x + 1 + 3x - 4 = 13$	3. substitution Prop.
4. $8x - 3 = 13$	4. Combine like terms
5. $8x = 16$	5. Addition Prop.
6. $x = 2$	6. Division Prop.
7. $BC = 3(2) - 4$	7. substitution Prop.
8. $BC = 2$	8. Simplify

Reminder { Same side = simplify / combine like terms  
 Both sides = Prop. of Equality (+, -, x, ÷)  
Guided Practice:

**Part A-** Identify the property that justifies each statement.

- $AB = AB$
- If  $m\angle 1 = m\angle 2$  and  $m\angle 2 = m\angle 4$ , then  $m\angle 1 = m\angle 4$
- If  $x = y$ , then  $y = x$ .
- If  $ST = YZ$ , and  $YZ = PR$ , then  $ST = PR$
- If  $KL = PR$ , then  $KL - AB = PR - AB$
- $412 = 412$
- If  $b = a$  and  $b = 0$ , then  $a = 0$
- Figure A = Figure A
- If  $m\angle DEF = m\angle ABC$ , then  $m\angle DEF + m\angle GHI = m\angle ABC + m\angle GHI$
- If  $x = y$ , then  $\frac{x}{3} = \frac{y}{3}$
- If  $AB = CD$ , then  $CD = AB$
- If  $\frac{x}{2} = 7$ , then  $x = 14$
- If  $x = 5$  and  $b = 5$ , then  $x = b$
- If  $XY - AB = WZ - AB$ , then  $XY = WZ$
- If  $m\angle A = m\angle B$ , and  $m\angle B = m\angle C$ , then  $m\angle A = m\angle C$

Reflexive  
Transitive  
Symmetric  
Transitive  
Subtraction Prop.  
Reflexive  
Transitive  
Reflexive  
Addition Prop.  
Division Prop.  
Symmetric  
Multiplication  
Transitive  
Subtraction Prop.  
Transitive

**Part B-** Use the property to complete the statement.

- Reflexive Property: SE = SE
- Symmetric Property: If  $m\angle JKL = m\angle RST$ , then  $m\angle RST = m\angle JKL$
- Transitive Property:  $m\angle F = m\angle J$  and  $m\angle J = m\angle L$ , then  $m\angle F = m\angle L$
- Addition Property: If  $RS = TU$ , then  $RS + 20 =$   $TU + 20$
- Multiplication Property: If  $m\angle 1 = m\angle 2$  then  $3(m\angle 1) =$   $3(m\angle 2)$
- Substitution Property: If  $a = 20$ , then  $5a =$   $5(20)$

**Part C-** Complete the two-column proofs below using the appropriate properties.

22. Given:  $x = \frac{1}{7}y - 9$   
 Prove:  $y = 7x + 63$

	Statement	Reason
1.	$x = \frac{1}{7}y - 9$	1. given
2.	$x + 9 = \frac{1}{7}y$	2. Addition Prop.
3.	$7(x + 9) = y$	3. Mult. Prop.
4.	$7x + 63 = y$	4. Distributive Prop.
5.	$y = 7x + 63$	5. Symmetric

23. Given:  $5(x - 3) = 4(x + 2)$   
 Prove:  $x = 23$

	Statement	Reason
1.	$5(x - 3) = 4(x + 2)$	1. Given
2.	$5x - 15 = 4x + 8$	2. Distribute Prop.
3.	$x - 15 = 8$	3. Subtraction Prop.
4.	$x = 23$	4. Add. Prop.