

Unit 1: Basics of Geometry and Proofs

After completion of this unit, you will be able to...

Learning Target #1: Recognize and Use Geometric Segment and Angle Relationships

- Name angles and lines
- Define and recognize the following relationships
 - Complementary and Supplementary Angles
 - Linear Pair
 - Vertical Angles
 - Midpoint
 - Angle and Segment Bisector
 - Angle Addition and Segment Addition
 - Perpendicular Lines
 - Parallel Line Relationships (Alt. Int, Alt. Ext, Consecutive Int., Consecutive Ext., Corresponding)
- Use the relationships to find missing segment lengths and angles


Learning Target #2: Algebraic and Geometric Proof

- Prove algebraically a geometric relationship using a two column proof
- Prove theorems about lines and angles using a two column proof
- Prove theorems about parallel lines using a two column proof

Basics of Geometry

Naming Angles and Lines


Point



A


Points are named with capital letters.

Line Segment



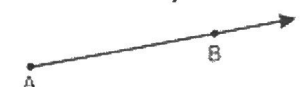
Two points are connected with a straight line. This line segment can be named \overline{AB} or \overline{BA} .

Line



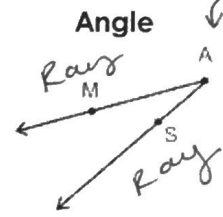
A line does not have a beginning or end point. Lines are named using two points on the line. This line can be named \overleftrightarrow{VW} or \overleftrightarrow{WV} .

Ray



Rays start with a point but continue to infinity in one direction. Rays are named using its starting point and one other point on the ray. The ray can be named \overrightarrow{AB} but NOT \overrightarrow{BA} .

Angle *Vertex*



Angles are made up of two rays that have the same beginning point. The point is called the vertex and the two rays are called the side of the angle. Angles can be name in ways:

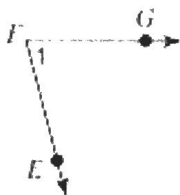
One Letter (if the vertex is not shared): $\angle A$

Number (if given): $\angle 1$

Three Letters (vertex is middle letter): $\angle MAS$ or $\angle SAM$

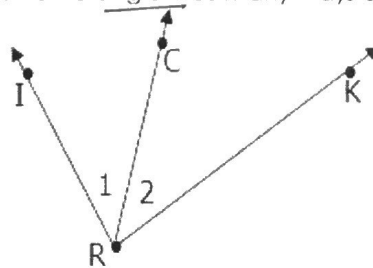
Rays \overrightarrow{AM} , \overrightarrow{AS}

a. Name the angle in four ways:



- 1) $\angle 1$
- 2) $\angle F$
- 3) $\angle GF\overline{E}$
- 4) $\angle \overline{E}F\overline{G}$

b. Name angle 1 as many ways as possible:



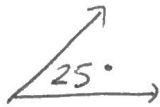
- 1) $\angle 1$
- 2) $\angle \overline{I}R\overline{C}$
- 3) $\angle \overline{C}R\overline{I}$

not $\angle R$ (Shared vertex of another angle)

TYPES OF ANGLES

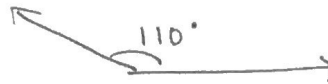
Acute Angles

Acute angles have measures between 0° & 90°



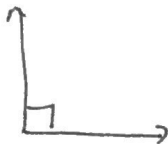
Obtuse Angles

Obtuse Angles have measures between 90° & 180°



Right Angles

Right Angles measure exactly 90°



Straight Angles

Straight Angles measure exactly 180°



Important Geometry Symbols

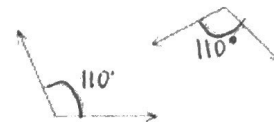
\angle Angle

\triangle Triangle

Congruent Angles

\cong Congruent (same shape & size)

$^\circ$ Degrees



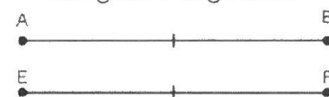
\perp Perpendicular (90 degrees)

m Measure of

\parallel Parallel

\sim Similar

Congruent Segments



Using Geometry Terminology

A **conditional statement** (if-then) is a statement that contains a hypothesis (if) and conclusion (then).

Ex. **If** a student plays basketball, **then** they are an athlete.

A **converse** is a statement that has the hypothesis and conclusion switched around.

Ex. **If** a student is an athlete, **then** they play basketball. (Is this true?)

* A **postulate** is a statement that is accepted as true without proof.

* A **theorem** is a statement that must be proven before it can be accepted as true. We are going to prove many theorems throughout this unit. We will prove a few of the following relationships on Day 3.

Practice: Take the following statement: *I do my homework; I get my allowance*, and write it in if-then form and then write the converse of it.

If I do my homework, then I get my allowance. 3

If I get my allowance, then I do my homework.

Supplementary and Complementary Angles

Complementary Angles: Two or more angles whose sum of measures equals 90° .

40° and 50° angles are complementary angles because $40^\circ + 50^\circ = 90^\circ$.

Example: A 30° angle is called the complement of the 60° angle.
Similarly, the 60° angle is the complement of the 30° angle.

$$\begin{aligned} 30^\circ + 60^\circ &= 90^\circ \\ 60^\circ + 30^\circ &= 90^\circ \end{aligned}$$

Practice: Find the **complement** of each angle.

a. 35°

$$\begin{aligned} x + 35 &= 90 \\ x &= 55^\circ \end{aligned}$$

b) Two angles, $2x^\circ$ and $3x^\circ$ are complementary. Find the value of x and each angle.

$$\begin{aligned} 2x + 3x &= 90 & 2(18) &= 36^\circ \\ 5x &= 90 & 3(18) &= 54^\circ \\ x &= 18^\circ \end{aligned}$$

Check
 $36 + 54 = 90$

Supplementary Angles: Two or more angles whose sum of measures equals 180° .

60° and 120° angles are supplementary angles because $60^\circ + 120^\circ = 180^\circ$.

Example: A 70° angle is called the supplement of the 110° angle.
Similarly, the 110° angle is the supplement of the 70° angle.

$$\begin{aligned} 70 + 110 &= 180 \\ 110 + 70 &= 180 \end{aligned}$$

Practice: Find the **supplement** of each angle.

a.) 126°

$$\begin{aligned} x + 126 &= 180 \\ x &= 54^\circ \end{aligned}$$

b) Two angles, $4x^\circ$ and $6x^\circ$ are supplementary. Find the value of x and each angle.

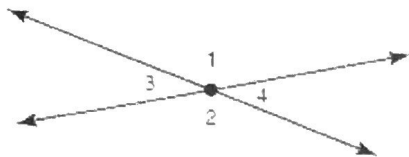
$$\begin{aligned} 4x + 6x &= 180 & 4(18) &= 72^\circ \\ 10x &= 180 & 6(18) &= 108^\circ \\ x &= 18^\circ \end{aligned}$$

Check
 $72 + 108 = 180$ 4

Special Pairs of Angles

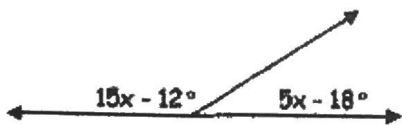
Linear Pair: Two adjacent (next to) angles whose noncommon sides are opposite rays. A linear pair also forms a line (supplementary).

a. Name all the linear pairs in the diagram below:



$$\begin{aligned} \angle 1 + \angle 4 &= 180 \\ \angle 3 + \angle 1 &= 180 \\ \angle 3 + \angle 2 &= 180 \\ \angle 2 + \angle 4 &= 180 \end{aligned}$$

b. Solve for x:



$$15x - 12 + 5x - 18 = 180$$

$$20x - 30 = 180$$

$$20x = 210$$

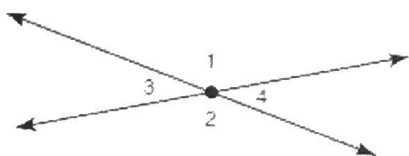
$$x = 10.5$$

Check

$$\begin{aligned} 15(10.5) - 12 &= 145.5^\circ \\ 5(10.5) - 18 &= 34.5^\circ \end{aligned}$$

Vertical Angles: Two nonadjacent angles that are formed by two intersecting lines. Vertical angles are congruent.

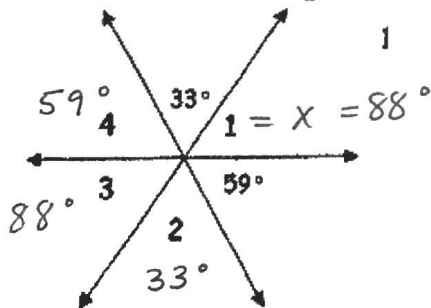
a. Name all the vertical angles in the diagram below:



$$\angle 1 \cong \angle 2$$

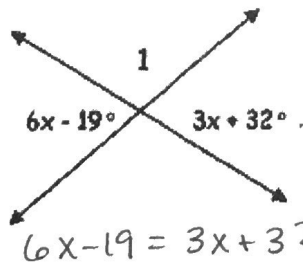
$$\angle 3 \cong \angle 4$$

b. Find the measure of angles 1, 2, 3, and 4.



$$\begin{aligned} 59 + 33 + x &= 180 \\ x &= 88^\circ \end{aligned}$$

c. Solve for x. Then determine the measure of angle 1.



$$\begin{aligned} 6x - 19 &= 3x + 32 \\ 3x - 19 &= 32 \\ 3x &= 51 \\ x &= 17 \end{aligned}$$

$$6(17) - 19 = 83$$

$$\begin{aligned} 180 - 83 &= 97^\circ \\ m\angle 1 &= 97^\circ \end{aligned}$$