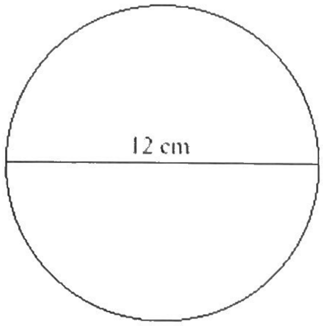
Warm-Up

For problems 1-2, find the area of the given figure.

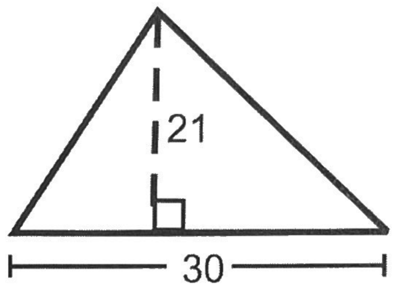
1.



$A = \pi r^2 = \pi(6)^2$

$\boxed{36\pi \text{ cm}^2}$

2.



$A = \frac{1}{2}bh$

$\frac{1}{2}(30)(21) = \boxed{315}$

Solve for *x*. When necessary, round to hundredths place and leave pi in answers that contain it.

3. $x = \frac{1}{\cancel{3}}(2)(\overset{4}{\cancel{12}})(5)$

$(8)(5)$

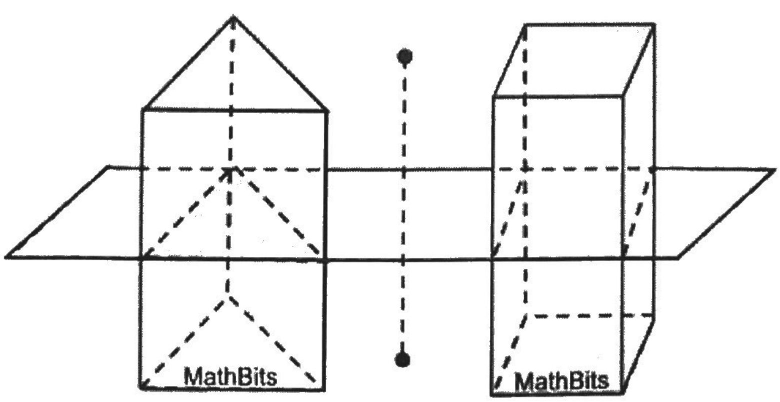
$\boxed{40}$

4. $x = \frac{1}{3}\pi(5)^2(22)$

$\boxed{\frac{550\pi}{3}}$

Cross Sections

Imagine a plane slicing through a solid. The intersection of the plane and the solid is called a cross section. For example, the diagram shows that an intersection of a plane and a triangular pyramid is a triangle. Also, an intersection of a plane in a rectangular prism is a rectangle.

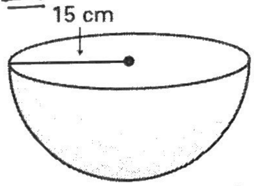
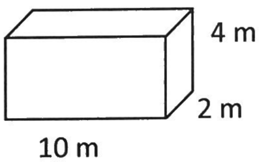
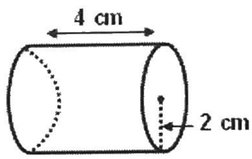
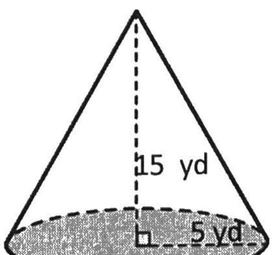
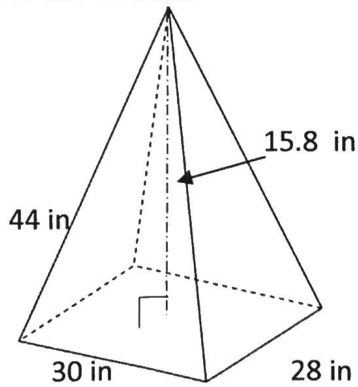


Triangular Base

Square/rectangle Base

Learning Task: Volumes of Cylinders, Cones, Pyramids, and Spheres

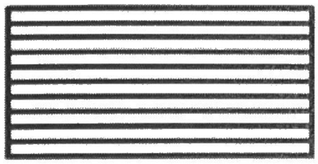
Volume Formulas Graphic Organizer

Shape	Formula	Example 1	Example 2
Sphere	$V = \frac{4}{3}\pi r^3$	A beach ball has a diameter of 8 inches. Find its volume. $r = 4$ $V = \frac{4}{3}\pi(4)^3$ $\boxed{\frac{256\pi}{3} in^3}$	Find the volume of the hemisphere.  $\boxed{2250\pi cm^3}$ $\frac{1}{2}\cdot\frac{4}{3}\pi(15)^3 = \frac{4500\pi}{2}$
Find the volume of prisms and cylinders.	V=Bh (where B is the area of the base) $A_{Rectangle}$= bh A_{Circle}= πr^2	Find the volume.  $V = 10(2)(4)$ $\boxed{80 m^3}$	Find the volume.  $V = \pi r^2 h$ $V = (2)^2(4)\pi$ $\boxed{16\pi cm^3}$
Find the volume of pyramids and cones.	$V = \frac{1}{3}Bh$ (where B is the area of the base)	Find the volume.  $V = \frac{1}{3}\pi r^2 h$ $\frac{1}{3}\pi(5)^2(15) = \frac{375\pi}{3}$ $\boxed{125\pi yd^3}$	Find the volume.  $V = \frac{1}{3}(30)(28)(15.8)$ $\boxed{4424 in^3}$

Learning Task: Cavaleri's Principle

Materials: 10 quarters

In order to explore Cavalieri's Principle construct a right cylinder using the 10 quarters.



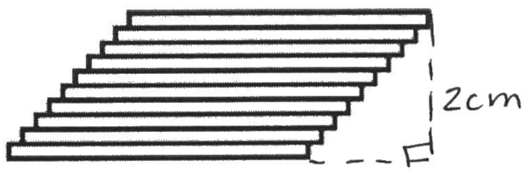
Recall that the volume of a cylinder is $V = Bh$. The height of the stack of quarters is 2 cm.

1. The diameter of a quarter is 2.4 cm. Calculate the area of the circular base.

   $r = 1.2$ $\quad \pi r^2 = \pi (1.2)^2 = 1.44\pi$

2. Find the volume of the cylinder. $V = 2(1.44\pi) = 2.88\pi\ cm^3$

Now we are going to create an oblique cylinder. Slightly move the quarters to form an oblique cylinder like the image below.



3. Does the height of your right cylinder change? Does the area of the base change? Does the volume change?

   – No
   – No
   – No !!

4. Look at the bases of all of your quarters. What two things do you notice?

   - Same Base
   - Same Height

The bases of the quarters are also referred to as **cross sections

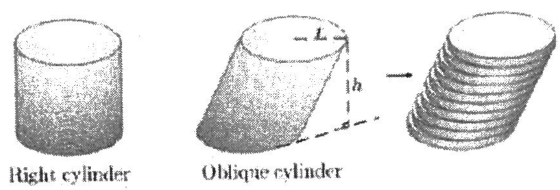
5. What can we conclude about the area of each cross section?

They are the same

6. Create a conjecture about the volume of a right and oblique cylinder with congruent bases and heights.

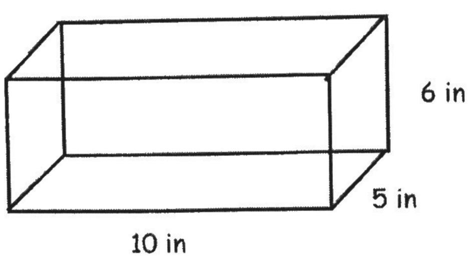
Volumes of Right + Oblique Cylinder are Congruent w/ congruent bases + heights.

Cavalieri's Principle States: The volumes of two solids are equal if the areas of corresponding sections drawn parallel to some given plane are equal.



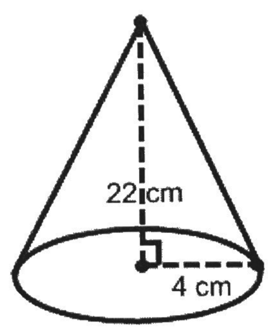
Guided Practice: Use volume formulas to calculate the volume of the figure described in the following problems. Leave answers in terms of pi or the nearest hundredth.

1.



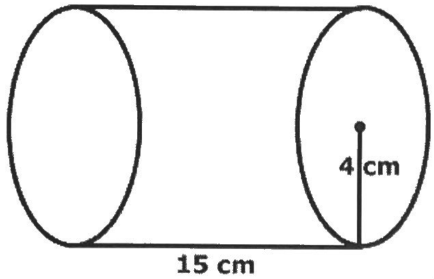
$V = 10(5)(6) = 300\,in^3$

2.



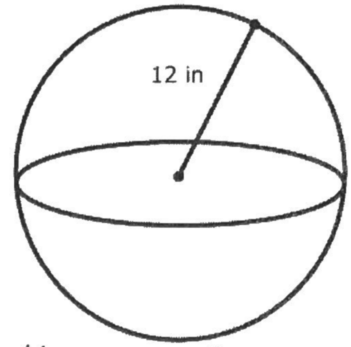
$\frac{1}{3}(\pi(4)^2)(22) = \frac{352\pi}{3}\ cm^3$

3.



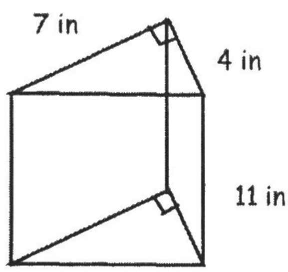
$V = \pi(4)^2(15) = 240\pi \text{ cm}^3$

4.



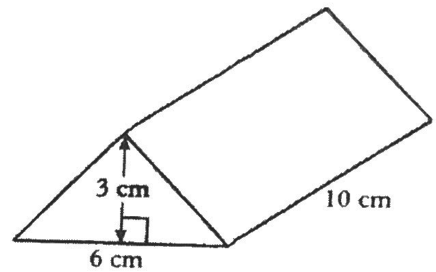
$V = \frac{4}{3}\pi(12)^3 = 2304\pi \text{ in}^3$

5.



$\frac{1}{2}(4)(7)(11) = 154 \text{ in}^3$

6.



$\frac{1}{2}(6)(3)(10)$

90 cm^3

7. The volume of a cylindrical watering can is 100cm^3. If the radius is doubled, then how much water can the new can hold?

$V = \pi r^2 h$

$100 = \pi r^2$

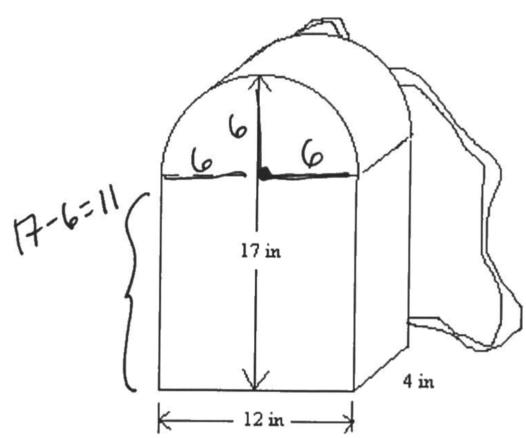
$V = \pi(2r)^2 h$

$2^2 r^2$

$V = \pi \, 4 r^2 h$

4 times as much

8. Approximate the volume of the backpack that is 17 in. x 12 in. x 4 in. The top of the backpack is half a cylinder and the bottom of the backpack is a rectangular prism.



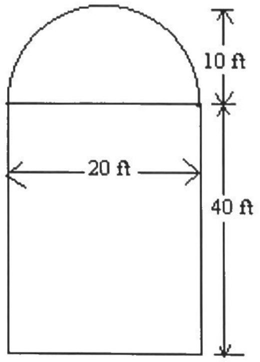
Bottom: $(12)(4)(11) = 528$

Top: $\pi(6)^2(4) = \frac{144\pi}{2} = 72\pi$

$528 + 72\pi$

754.19 in^3

9. Find the volume of the Grain Silo shown below that has a diameter of 20 ft. and a height of 50 ft. The top of the Grain Silo is a hemisphere and the bottom of the silo is a cylinder.



$$\text{Top} = \frac{\frac{4}{3}\pi(10)^3}{2} = \frac{2000}{3}\pi$$

$$\text{Bottom} = \pi(20)^2(40) = 16{,}000\pi$$

$$\frac{50{,}000}{3}\pi$$

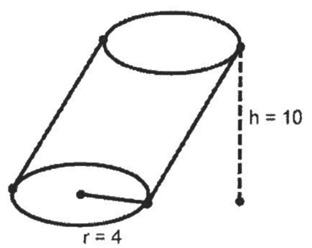
$$\approx 52{,}359.88 \text{ ft.}^3$$

10. The diameter of a baseball is about 1.4 in. How much rubber is needed to fill it?

$r = .7$ $\quad V = \frac{4}{3}\pi(.7)^3 = \frac{343}{750}\pi \approx 1.44 \text{ in}^3$

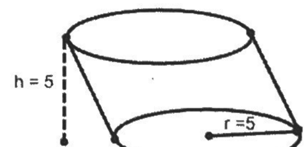
Calculate the volume of the cylinder pictured in problems 11-12.

11.



$V = \pi(4)^2(10) = 160\pi \text{ units}^3$

12.

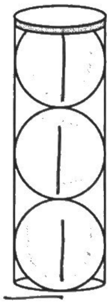


$V = \pi(5)^2(5) = 125\pi \text{ units}^3$

13. A sphere of ice cream is placed onto your ice cream cone. Given that the cone has a diameter of 8 centimeters, find the volume of the ice cream.

$r = 4\text{cm}$ $\quad \text{Sphere} = \frac{4}{3}\pi(4)^3 = \frac{256\pi}{3}$

14. Tennis balls with a diameter of 2.5 in are sold in cans of three. The can is a cylinder. What is the volume of the space **not** occupied by tennis balls? Assume the balls touch the can on the sides, top, and bottom.



2.5
+
2.5
+
2.5

7.5=h

r=1.25

Cylinder $V = \pi(1.25)^2(7.5) = \frac{375\pi}{32}$

Sphere (x3) $V = 3 \times \frac{4}{3}(1.25)^3\pi = \frac{125\pi}{16}$

$\frac{125}{32}\pi \text{ in}^3$

Skills Practice: Volume and Cavalieri's Principle

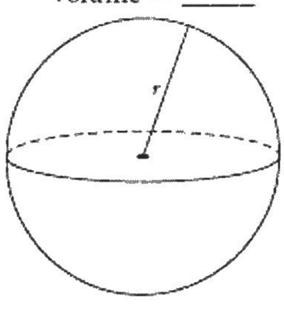
Directions: Find the volume for each of the solids pictured or described in the following problems. Leave answers in terms of pi or round to the nearest hundredth.

1. Find the volume of a sphere when the diameter is 24 cm. $r=12$

$V = \frac{4}{3}(12)^3\pi = 2304\pi \text{ cm}^3$

2. $r = 9$ yd

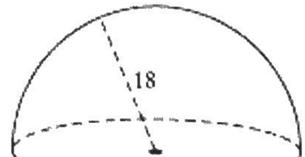
Volume = _____



$V = \frac{4}{3}(9)^3\pi$

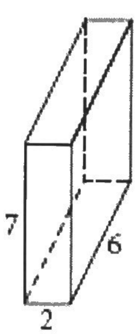
$972\pi \text{ yd}^3$

3. Find the volume of the hemisphere.



$V = \frac{\frac{4}{3}(18)^3\pi}{2} = 3888\pi \text{ units}^3$

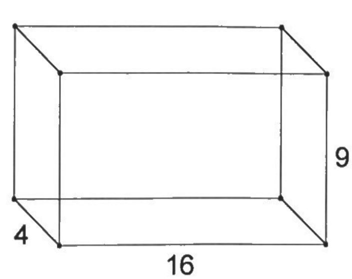
4.



$V = 2(6)(7)$

84 units^3

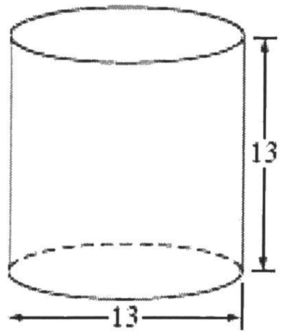
5.



$V = 4(16)(9)$

576 units^3

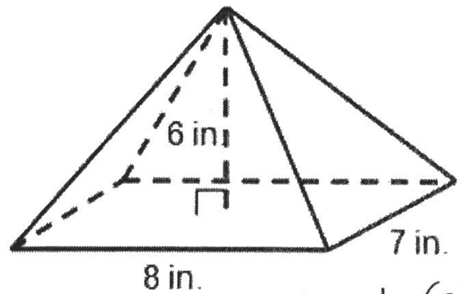
6.



$V = \pi(6.5)^2(13)$

$\frac{2197}{4}\pi$ units^3

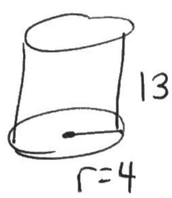
7.



$V = \frac{1}{3}(8)(7)(6)$

112 in^3

8. Find the volume of a right cylinder with a radius of 4 ft. and a height of 13 ft.



$V = \pi(4)^2(13) = 208\pi \text{ ft.}^3$

9. Determine the volume of a right cylinder with a diameter of 16 in. and a height of 6 in.

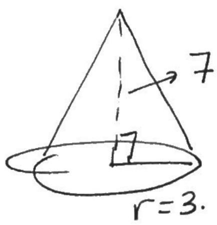
$r = 8$

$V = \pi(8)^2(6)$

$384\pi \text{ in}^3$

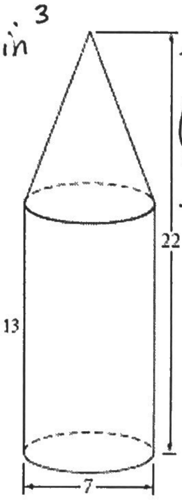
10. A right circular cone has a diameter of 6.2 in. and a height of 7 in. Calculate the volume of the cone.

$r = 3.1$



$V = \frac{1}{3}\pi(3.1)^2(7) = \frac{6727}{300}\pi \text{ in}^3$

11. Find the volume of the figure to the right.



$22 - 13$

$h = 9$

$r = 3.5$

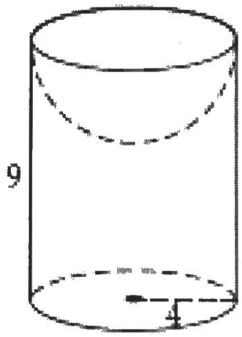
$\text{Cone} = \frac{1}{3}\pi(3.5)^2(9) = \frac{147}{4}\pi$

$+ \qquad +$

$\text{Cylinder} = \pi(3.5)^2(13) \quad \frac{637\pi}{4}$

$196\pi \approx 615.75$ units^3

12. Calculate the volume of cylinder with a hemisphere taken out of the top.



Cylinder

$\pi(4)^2(9) = 144\pi$

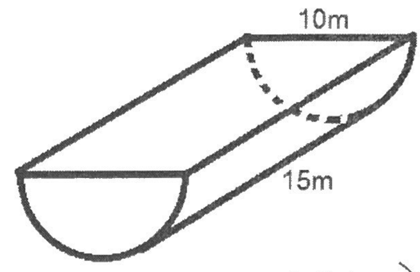
$-$

hemisphere

$\dfrac{\frac{4}{3}(4)^3\pi}{2} = \dfrac{128}{3}\pi$

$\dfrac{304}{3}\pi \text{ unit}^3$

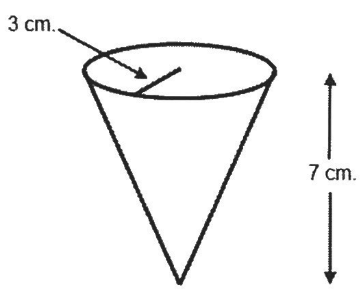
13. Calculate the volume.



$V = \frac{1}{2} \cdot \pi(5)^2(15)$

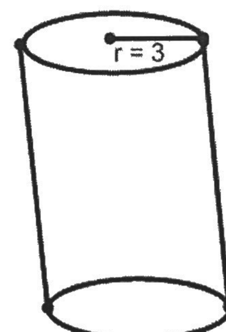
$\frac{375}{2}\pi \, m^3$

14. Find the volume of the cone below.



$V = \frac{1}{3}\pi(3)^2(7) = 21\pi \, cm^3$

15. Calculate the volume, given that the height is three times the length of the radius.



$3 \times 3 = 9$

$V = \pi(3)^2(9) = 81\pi \text{ unit}^3$

Volume Applications and Practice:

1. Given a cylinder has a volume of 950 cm^3 and a height of 12 cm, what is the radius?

$V = \pi r^2 h$

$950 = \pi r^2 (12)$

$\dfrac{950}{12\pi} = \dfrac{12\pi r^2}{12\pi}$

$25.199 = r^2$

$\sqrt{r^2} = \sqrt{25.199}$

$r \approx 5.02 \, cm$

2. Given a sphere has a volume of 325 ft^3, what is the radius of the sphere?

$V = \frac{4}{3}\pi r^3$

$\dfrac{325}{\frac{4}{3}\pi} = \dfrac{\frac{4}{3}\pi r^3}{\frac{4}{3}\pi}$

$\sqrt[3]{r^3} = \sqrt[3]{77.588}$

$r \approx 4.27 \, ft$

3. A rectangular prism has a volume of 624 ft^3, a length of 8ft and a height of 10ft. What is the width of the prism?

$V = L \cdot W \cdot H$

$624 = 8 \cdot w \cdot 10$

$\frac{624}{80} = \frac{80w}{80}$

$w \approx 7.8$ ft

4. A cylinder has a radius of 3cm and a volume of 1024π cm^3. What is the height of the cylinder?

$V = \pi r^2 h$

$1024\pi = \pi(3)^2 h$

$\frac{1024\pi}{9\pi} = \frac{9\pi h}{9\pi}$

$h \approx 113.78$cm

5. If a sphere **doubles** its radius, what effect will this have on the volume of the sphere?

$V = \frac{4}{3}(2r)^3\pi = 2^3r^3 \Rightarrow 8r^3$

Complete the sentence: The volume will be 8 times more/less than the original sphere.

6. If a cylinder **doubles** its radius, what effect will this have on the volume of the cylinder?

$V = \pi(2r)^2h = 2^2r^2 \Rightarrow 4r^2$

Complete the sentence: The volume will be 4 times more/less than the original cylinder.

7. If a cone **triples** its radius, what effect will it have on the volume of the cone?

$V = \frac{1}{3}\pi(3r)^2h \Rightarrow 3^2r^2 = 9r^2$

Complete the sentence: The volume will be 9 times more/less than the original cone.