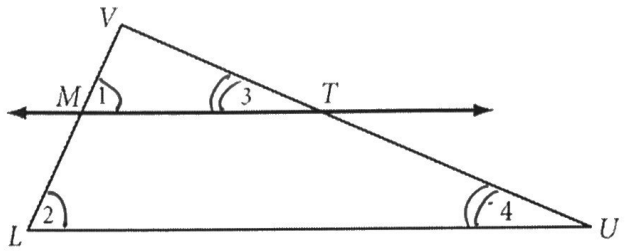
Triangle Proportionality Theorem

In the figure below, $\overleftrightarrow{MT} \| \overleftrightarrow{LU}$. Is $\triangle LUV$ similar to $\triangle MTU$? Yes, it is. A short proof can support this observation.

Given: $\triangle LUV$ with $\overleftrightarrow{MT} \| \overleftrightarrow{LU}$.

Prove: $\triangle LUV \sim \triangle MTV$



Statements	Reasons
$\overleftrightarrow{MT} \| \overleftrightarrow{LU}$.	Given
$\angle 1 \cong \angle 2$	Corresponding $\triangle$ Postulate
$\angle 3 \cong \angle 4$	Corresponding $\triangle$ Postulate
$\triangle LUV \sim \triangle MTV$	AA $\sim$

Triangle Proportionality Theorem:

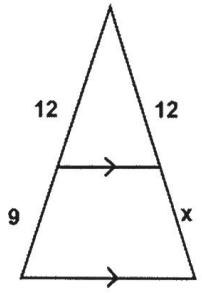
If a line parallel to one side of a triangle passes through the other two sides, then it divides the other two sides proportionally.

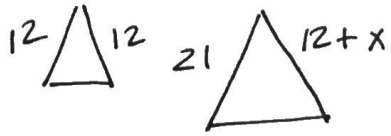
Conversely, if a line cuts two sides of a triangle proportionally, then it is parallel to the third side.

Guided Practice and Classwork:

Find the value of 'x' in each picture.

Ex 1:





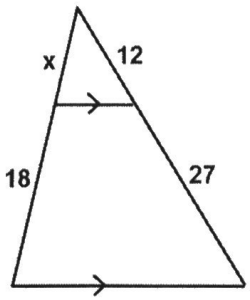
$\frac{12}{21} = \frac{12}{12+x}$

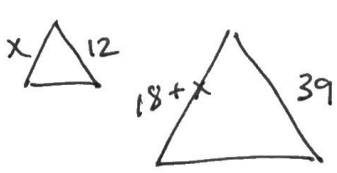
$144 + 12x = 252$

$12x = 108$

$x = 9$

Ex. 2:





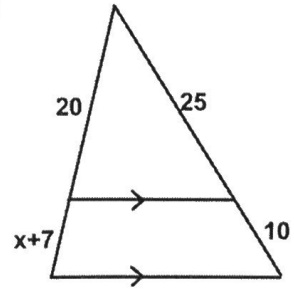
$\frac{x}{18+x} = \frac{12}{39}$

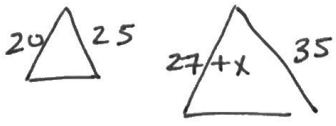
$39x = 216 + 12x$

$\frac{27x}{27} = \frac{216}{27}$

$x = 8$

Ex. 3:





$\frac{20}{27+x} = \frac{25}{35}$

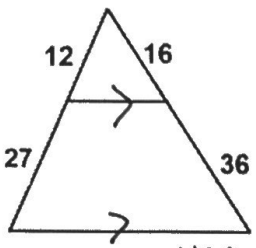
$700 = 675 + 25x$

$\frac{25}{25} = \frac{25x}{25}$

$1 = x$

Show that the given lines are parallel *(The Converse of this theorem)*

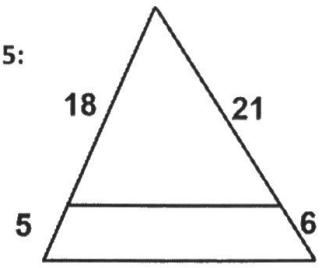
Ex 4:



$\frac{12}{39} = \frac{16}{52}$ yes

$624 = 624$ ✓

Ex. 5:

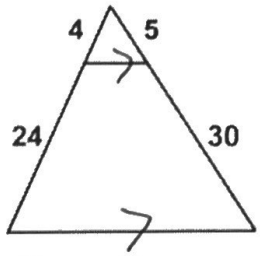


$\frac{18}{23} = \frac{21}{27}$

$486 \neq 483$

not parallel

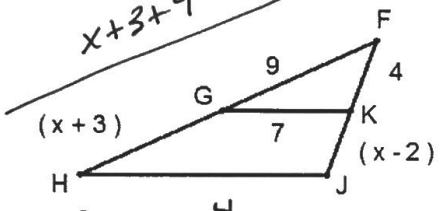
Ex. 6:



$\frac{4}{28} = \frac{5}{35}$ yes!

$140 = 140$ ✓

Ex. 7: Find the value of x if GK ∥ HJ.



$x+3+9 = x+12$

$4+x-2$

$x+2$

$\frac{9}{x+12} = \frac{4}{x+2}$

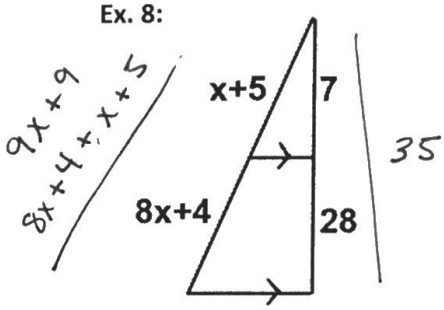
$9(x+2) = 4(x+12)$

$9x+18 = 4x+48$

$5x = 30$

$x = 6$

Ex. 8:



$\frac{9x+9}{x+5} = \frac{35}{7}$

$7(9x+9) = 35(x+5)$

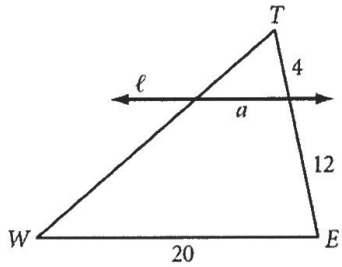
$63x+63 = 35x+175$

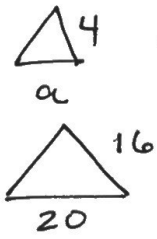
$28x = 112$

$x = 4$

Practice:

1. $l \parallel \overline{WE}$, a= 5



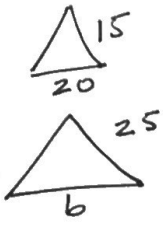


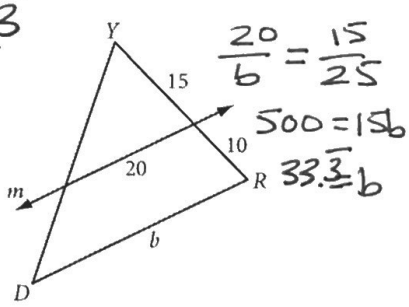
$\frac{4}{16} = \frac{a}{20}$

$80 = 16a$

$5 = a$

2. $m \parallel \overline{DR}$, b= $33.\overline{3}$



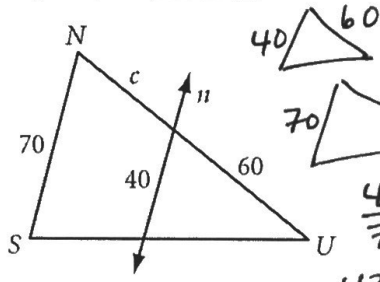


$\frac{20}{b} = \frac{15}{25}$

$500 = 15b$

$33.\overline{3} = b$

3. $n \parallel \overline{SN}$, c= 45



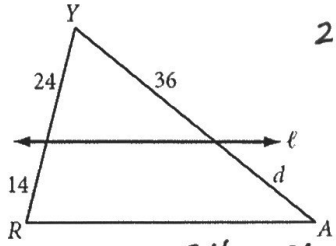
$\frac{40}{70} = \frac{60}{c+60}$

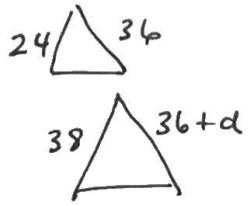
$4200 = 40c + 2400$

$1800 = 40c$

$45 = c$

4. $l \parallel \overline{RA}$, d= 21





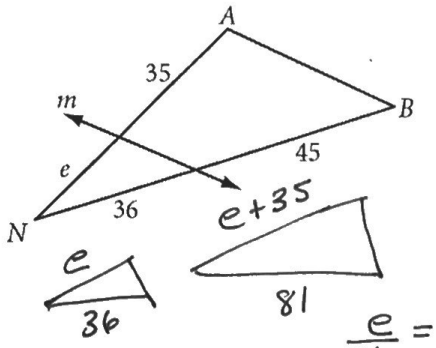
$\frac{24}{38} = \frac{36}{36+d}$

$864 + 24d = 1368$

$24d = 504$

$d = 21$

5. $m \parallel \overline{BA}$, e= 28

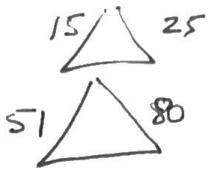


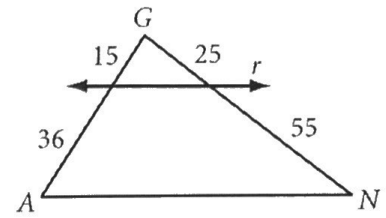
$\frac{e}{36} = \frac{e+35}{81}$

$81e = 36e + 1260$

$45e = 1260$

6. Is $r \parallel \overline{AN}$?





$\frac{15}{51} \neq \frac{25}{80}$

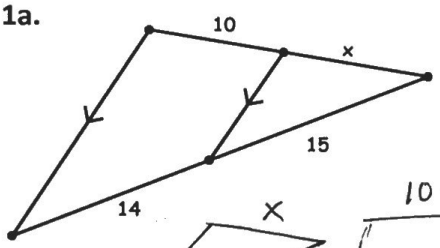
$1200 \neq 1275$

no, not parallel b/c not proportional

Skills Practice:

Use the Triangle Proportionality Theorem to find the missing value.

1a.



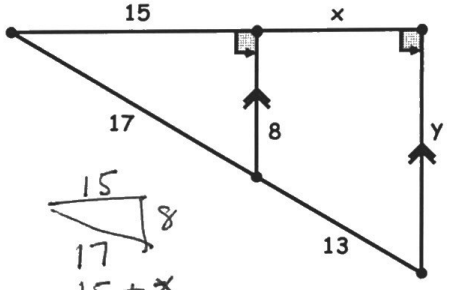
$\frac{x}{15} = \frac{10+x}{29}$

$29x = 150 + 15x$

$14x = 150$

$x = 10.71$

1b.



$\frac{15}{15+x} = \frac{17}{30}$

$450 = 17(15+x)$

$450 = 255 + 17x$

$195 = 17x$

$11.47 = x$

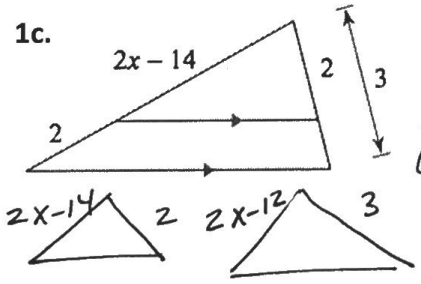
$\frac{17}{30} = \frac{8}{y}$

$17y = 30(8)$

$17y = 240$

$y = 14.12$

1c.



$\frac{2x-14}{2x-12} = \frac{2}{3}$

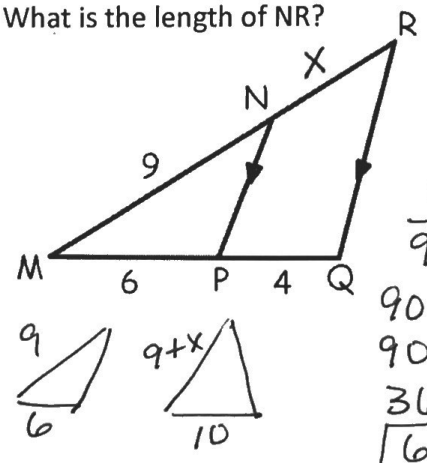
$3(2x-14) = 2(2x-12)$

$6x - 42 = 4x - 24$

$2x = 18$

$x = 9$

2. What is the length of NR?



$\frac{9}{9+x} = \frac{6}{10}$

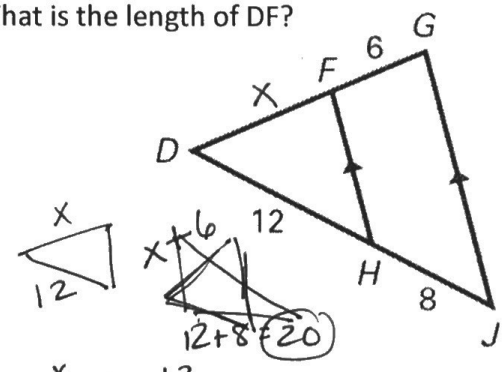
$90 = 6(9+x)$

$90 = 54 + 6x$

$36 = 6x$

$6 = x$

3. What is the length of DF?



$\frac{x}{x+6} = \frac{12}{20}$

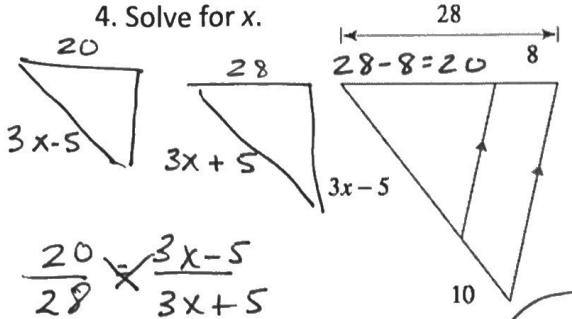
$20x = 12(x+6)$

$20x = 12x + 72$

$8x = 72$

$x = 9$

4. Solve for x.



$\frac{20}{28} = \frac{3x-5}{3x+5}$

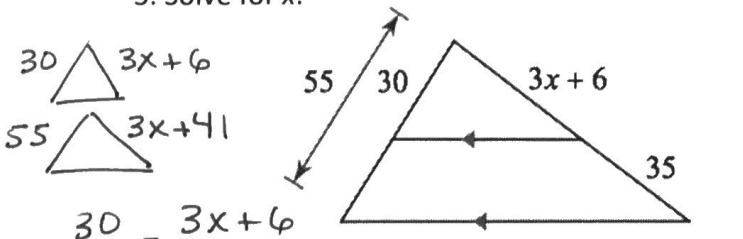
$20(3x+5) = 28(3x-5)$

$60x + 100 = 84x - 140$

$240 = 24x$

$x = 10$

5. Solve for x.



$\frac{30}{55} = \frac{3x+6}{3x+41}$

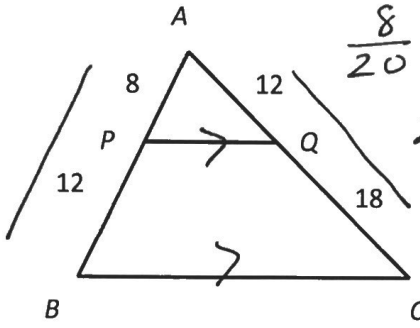
$30(3x+41) = 55(3x+6)$

$90x + 1230 = 165x + 330$

$900 = 75x$

$x = 12$

6. Is $\overline{PQ}$ parallel to $\overline{BC}$? Explain.

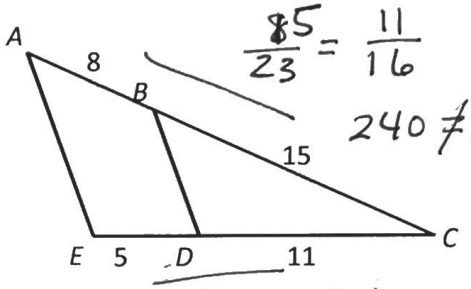


$\frac{8}{20} = \frac{12}{30}$

$240 = 240$

yes - Parallel

7. Is $\overline{AE}$ parallel to $\overline{BD}$? Explain.



$\frac{8}{23} = \frac{11}{16}$

$240 \neq 253$

No - Not Parallel blc not proportional

Angle Chaser: Find the measure of each lettered angle in the diagram below.