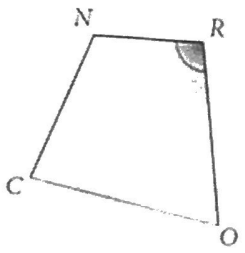
Lesson: Similar Triangles

Just as congruence introduced us to new notation, similarity will have its own set of notation.

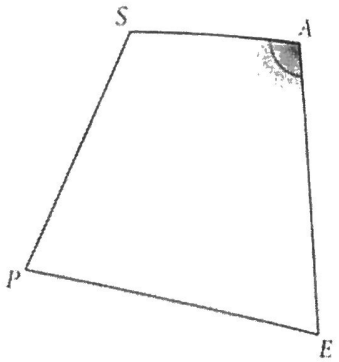
If ΔCAT is congruent to ΔMEW, we write $\Delta CAT \cong \Delta MEW$. If two polygons are *similar* we use the symbol that sits above the equal sign in the congruent symbol: [~].

For example, the statement CORN~ PEAS says that quadrilateral CORN is similar to quadrilateral PEAS. Just as in statements of congruence, the order of the letters tells you which segments and which angles in the two polygons correspond.



Corresponding angles are congruent:

Corresponding segments are proportional:



The ratio of the lengths of any two segments in one polygon is equal to the ratio of the corresponding two segments in the similar polygon. For example, $\frac{CO}{OR} = \frac{PE}{EA}$ or $\frac{NR}{CO} = \frac{SA}{PE}$

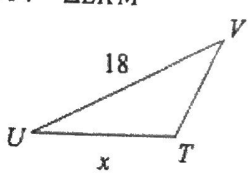
As we know, **Similar Figures** have the same shape, but not necessarily the same size.

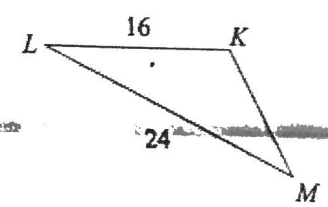
All corresponding angles are **equal** and corresponding sides are **proportional**. Proportionality is based on a scale factor which we will see later in transformation (dilations).

Guided Practice

Directions: Use the similarity statement to solve for x.

1. $\Delta UTV \sim \Delta LKM$

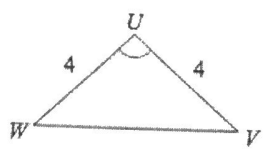


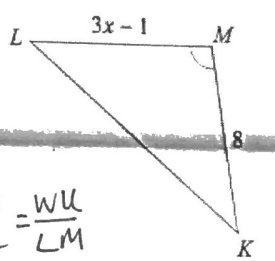


$\frac{UT}{LK} = \frac{TV}{KM} = \frac{VU}{ML}$

$\frac{x}{16} = \frac{18}{24}$ $24x = 288$ $x = 12$

2. $\Delta UVW \sim \Delta MKL$



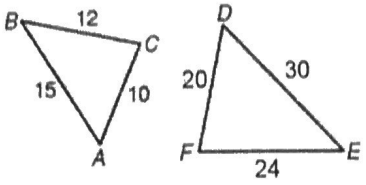


$\frac{UV}{MK} = \frac{VW}{KL} = \frac{WU}{LM}$

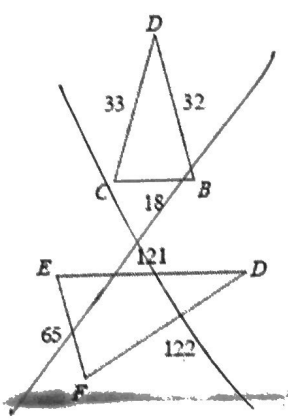
$\frac{4}{8} = \frac{4}{3x-1}$ $4(8) = 4(3x-1)$ $32 = 12x - 4$

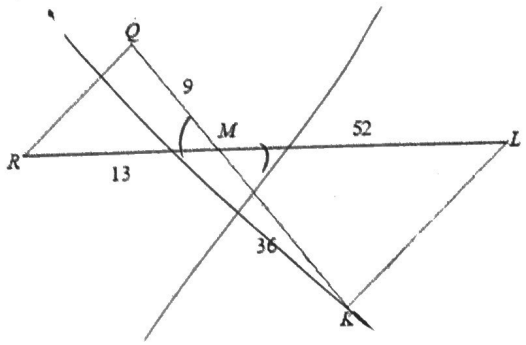
Proving Triangles are Similar Mathematically

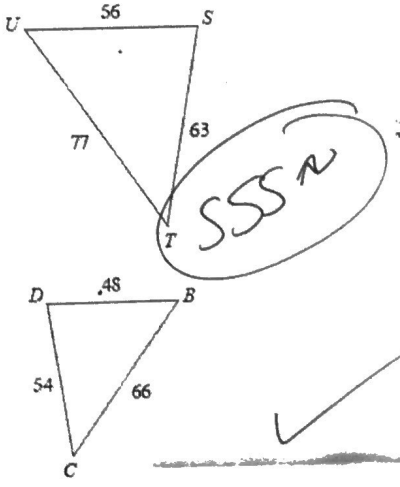
Complete the following graphic organizer:

Term/ Postulate	Definition/Explanation	Diagram
Side – Side - Side **SSS~**	all 3 corresponding sides are proportional.	 S M L $\frac{AC}{DF} = \frac{BC}{FE} = \frac{BA}{DE}$ $\frac{10}{20} = \frac{12}{24} = \frac{15}{30}$ $\frac{1}{2} = \frac{1}{2} = \frac{1}{2}$ ✓ 0.5 = 0.5 = 0.5 SSS~ **Statement:** $\triangle CAB \sim \triangle FDE$

Example: Which of the following example(s) can be shown triangles are similar by SSS~?





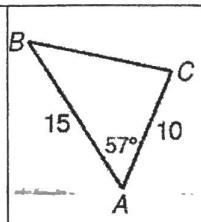
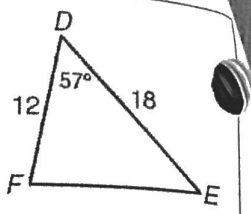


$\frac{18}{65} = \frac{32}{121} = \frac{33}{122}$

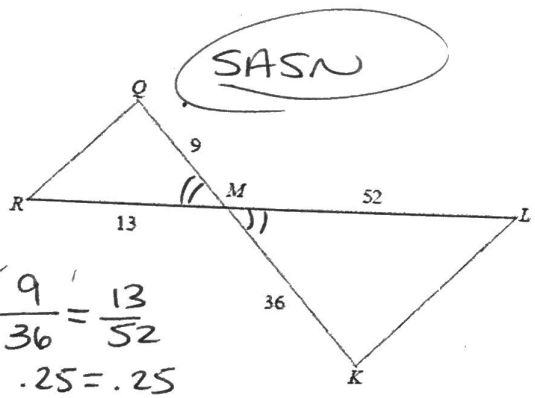
.2749 = .26

$\frac{56}{48} = \frac{63}{54} = \frac{77}{66}$

$1.1\overline{6} = 1.1\overline{6} = 1.1\overline{6}$

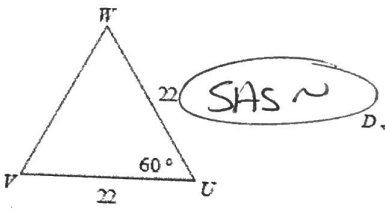
Side-Angle-Side **SAS~**	2 Corresponding proportional sides w/ an included $\cong$ angle.	  $\angle A \cong \angle D$ $\frac{AB}{DE} = \frac{AC}{DF}$ $\frac{10}{12} = \frac{15}{18}$ $.8\overline{3} \cong .8\overline{3}$ ✓ SAS~ **Statement:** $\triangle BCA \sim \triangle EFD$

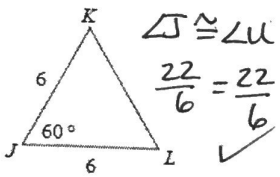
Example: **Which of the following example(s) can be shown triangles are similar by SAS~?**



✓ $\frac{9}{36} = \frac{13}{52}$

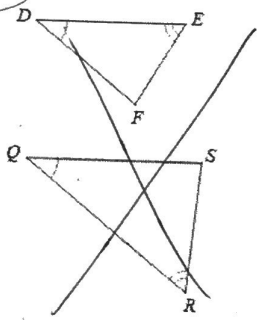
$.25 = .25$

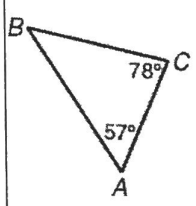
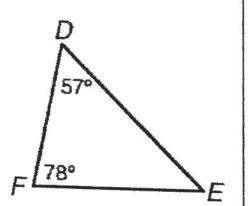




$\angle J \cong \angle U$

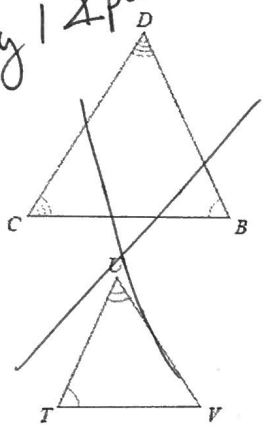
$\frac{22}{6} = \frac{22}{6}$ ✓

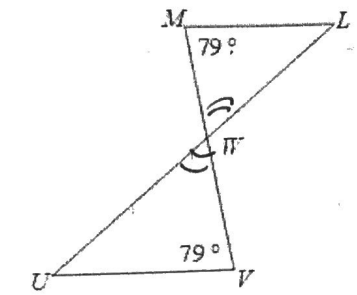


Angle- Angle **AA~**	2 congruent Corresponding angles	  $\angle A \cong \angle D$ $\angle C \cong \angle F$ **Statement:** $\triangle ABC \sim \triangle DEF$

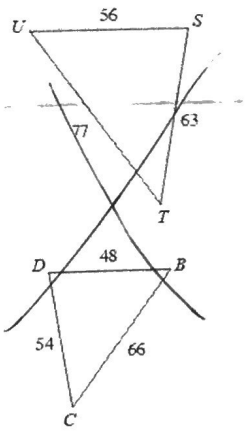
Example: Which of the following example(s) can be shown triangles are similar by AA~?

only 1 Δ pair ≅





∠M ≅ ∠V
∠MWL ≅ ∠VWU
AA~

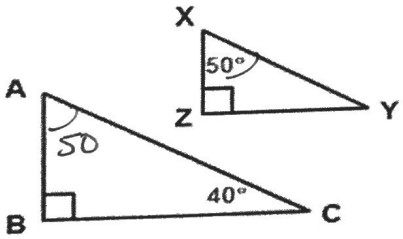


Guided Practice:

Directions: Show that the following triangles are similar, by showing either angles congruent or sides proportional. Next state the similarity statement.

1.

90
+40
130



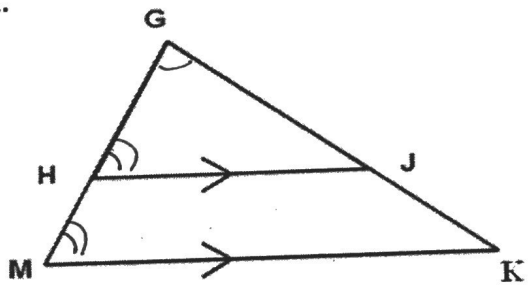
Show corresponding parts:

∠B ≅ ∠Z
∠A ≅ ∠X

Circle: AA~, SSS~, SAS~

Statement: △ABC ~ △XZY

2.



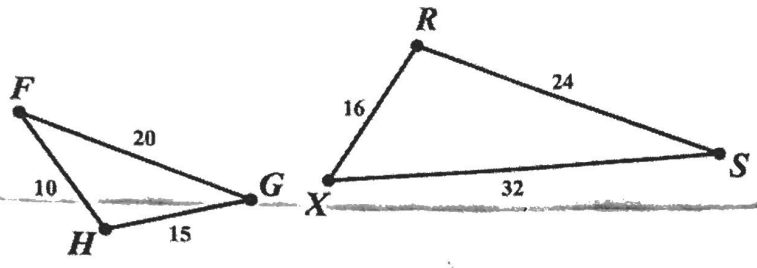
Show corresponding parts:

∠G ≅ ∠G Reflexive Prop.
∠GHJ ≅ ∠HMK Corresponding ∠'s

Circle: AA~, SSS~, SAS~

Statement: △GHJ ~ △GMK

3.



Show corresponding parts:

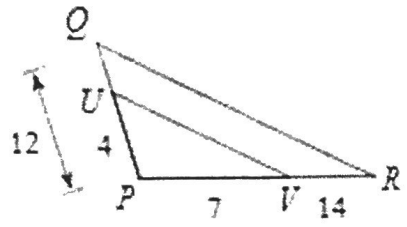
Small	Medium	Large
$\frac{10}{16}$	$\frac{15}{24}$	$\frac{20}{32}$
.625	.625	.625

Circle: AA~, SSS~, SAS~

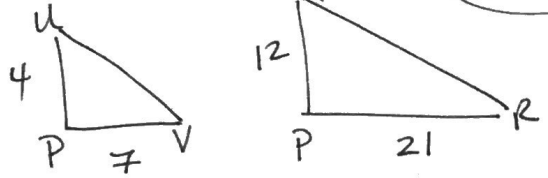
Statement: $\triangle FHG \sim \triangle XRS$

4. Determine if the two triangles are similar. If so, state the similarity statement and state how you know they are similar (SSS~, SAS~, AA~). Justify your conclusions.

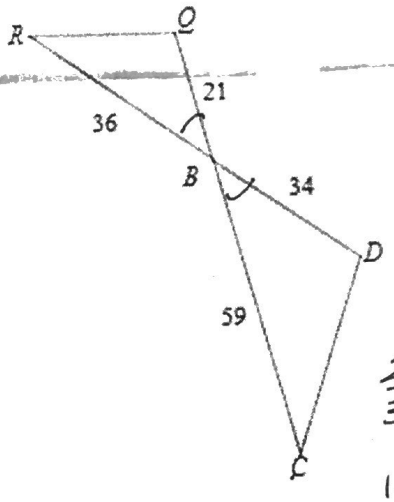
a.



$\triangle PQR \sim \triangle PUV$ SAS~



$\frac{4}{12} = \frac{7}{21}$ $\angle P \cong \angle P$ ✓

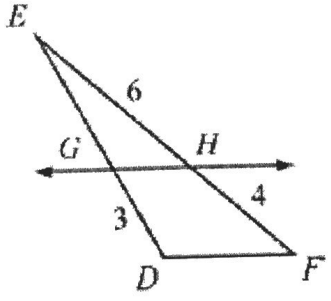


$\frac{21}{34} = \frac{36}{59}$

$1239 \neq 1224$

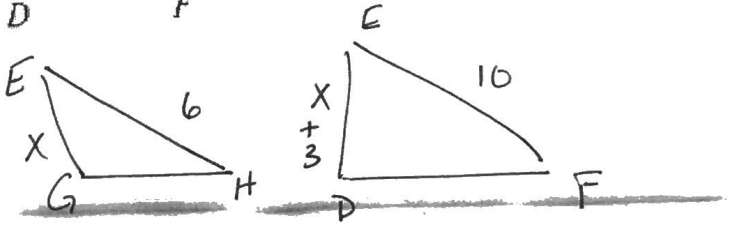
$\triangle BCD \sim$ not similar

5. Multiple Choice: In the triangle shown, $\overline{GH} \parallel \overline{DF}$



What is the length of $\overline{GE}$?

A. 2.0
B. 4.5
C. 7.5
D. 8.0

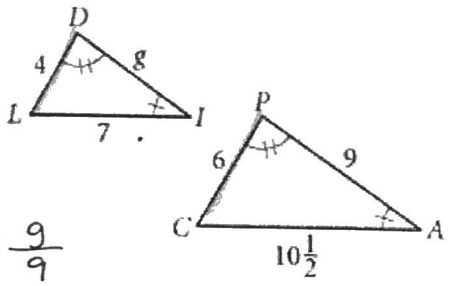


$\frac{X}{X+3} = \frac{6}{10}$

Homework- Similar Triangles

For questions 1 – 4, write a similarity statement. Then find the measures of the missing sides.

1. $g =$? 6

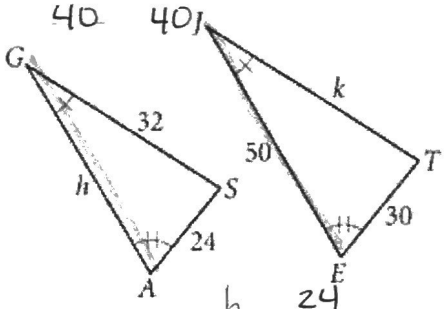


$\frac{4}{6} = \frac{9}{g}$

$36 = 6g$

$g = 6$

2. $h =$? 40, $k =$? 40



$\frac{h}{50} = \frac{24}{30}$

$h = 40$

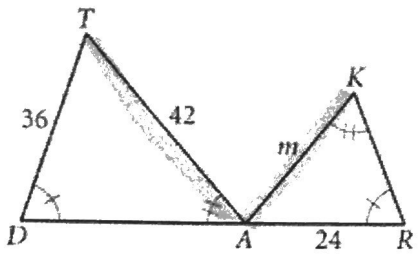
$1200 = 30h$

$\frac{32}{K} = \frac{24}{30}$

$K = 40$

$960 = 24K$

3. $m =$? 28



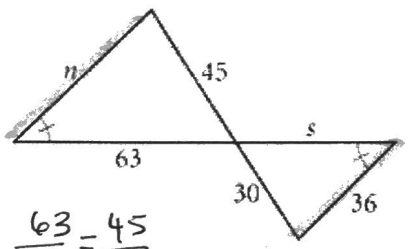
$\frac{36}{24} = \frac{42}{m}$

$1008 = 36m$

$m = 28$

4. $n =$? 54

$s =$? 42



$\frac{63}{s} = \frac{45}{30}$

$1890 = 45s$

$s = 42$

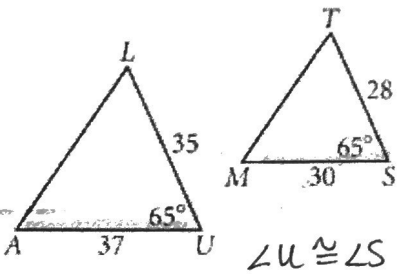
$\frac{45}{30} = \frac{n}{36}$

$1620 = 30n$

$n = 54$

5. Is $\Delta AUL \sim \Delta MST$?

Explain why or why not.



$\angle U \cong \angle S$

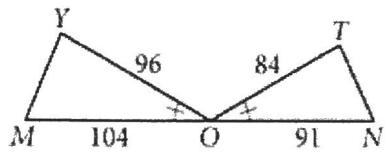
$\frac{37}{30} = \frac{35}{28}$

$1036 \neq 1050$

no
not SAS~

6. Is $\Delta MOY \sim \Delta NOT$?

Explain why or why not.



$\angle NOY \cong \angle NOT$

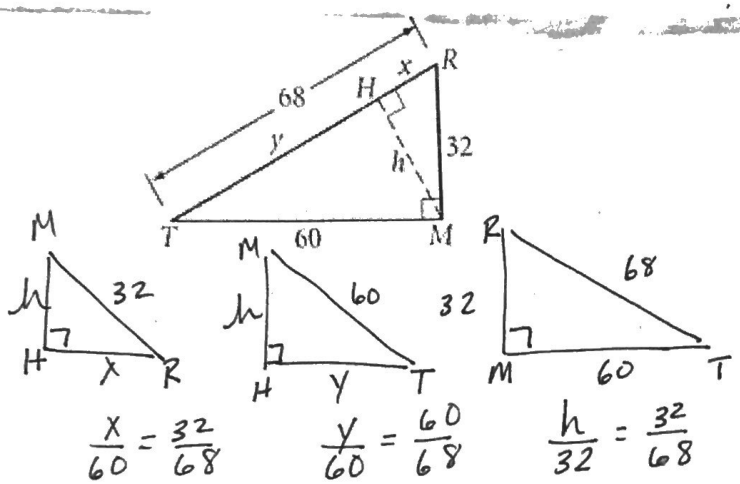
$\frac{96}{84} = \frac{104}{91}$

$8736 = 8736$

yes! SAS~

7. Why is $\Delta TMR \sim \Delta THM \sim \Delta MHR$?

Find x, y, and h.



$\frac{x}{60} = \frac{32}{68}$ $\frac{y}{60} = \frac{60}{68}$ $\frac{h}{32} = \frac{32}{68}$

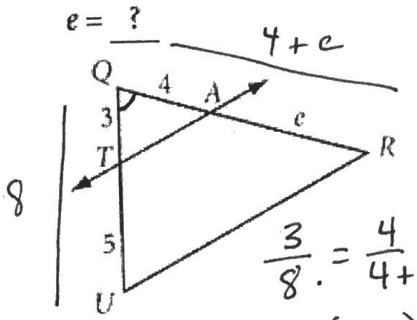
8. $\overline{TA} \| \overline{UR}$

Is $\angle QTA \cong \angle TUR$? yes! corresponding ∠s

Is $\angle QAT \cong \angle ARU$? yes!

Why is $\Delta QTA \sim \Delta QUR$? AA ~

$e = ?$



$\frac{3}{8} = \frac{4}{4+e}$

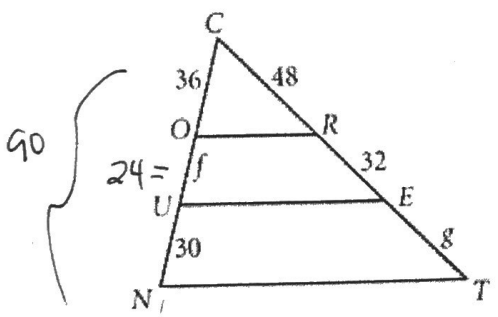
$3(4+e) = 32$

$12 + 3e = 32$

$\frac{3e}{3} = \frac{20}{3}$

9. $\overline{OR} \| \overline{UE} \| \overline{NT}$

$f = ?$, $g = ?$



$\frac{36}{36+f} = \frac{48}{80}$

$2880 = 48(36+f)$

$2880 = 1728 + 48f$

$1152 = 48f$

$f = 24$

$\frac{36}{90} = \frac{48}{80+g}$

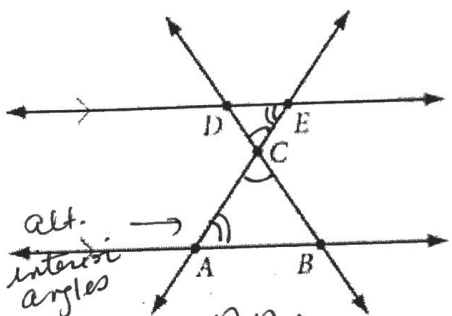
$36(80+g) = 4320$

$2880 + 36g = 4320$

$36g = 1440$

$g = 40$

10. $\Delta ABC \sim \Delta???$. Why?

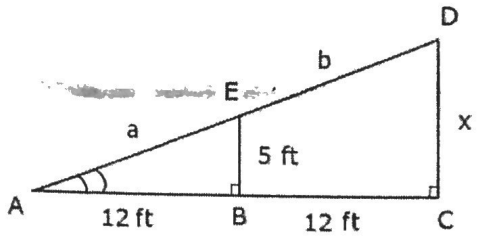


AA ~

$\Delta ABC \sim \Delta EDC$

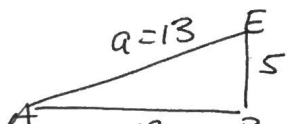
11. Use the figure below to answer the following questions.

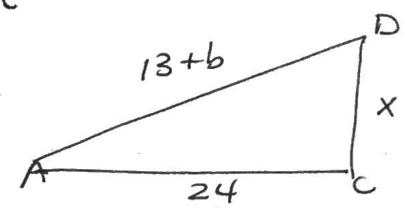
$5^2 + 12^2 = a^2$
$169 = a^2$
$13 = a$



a. $\Delta ABE \sim \Delta ACD$ by AA~

b. Solve for x.





$\frac{12}{24} = \frac{5}{x}$

$12x = 120$

$x = 10$

c. Solve for b given that a = 13.

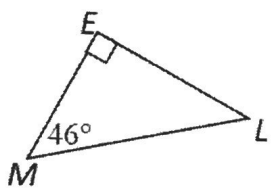
$\frac{12}{24} = \frac{13}{13+b}$

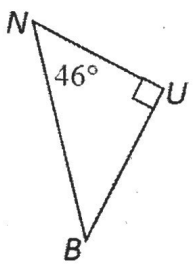
$156 + 12b = 312$

$12b = 156$

$b = 13$

12. Which of the following is true about the triangles below?

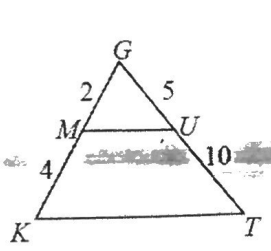


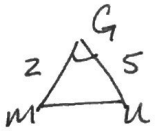


A. Similar but not congruent
B. Congruent but not similar
C. Both similar and congruent
D. Neither similar nor congruent

no sides are given so we don't know the sides are ~.
by AA~ we know Δ's are similar

13. Which of the following is true about the triangles below?





$\frac{2}{6} = \frac{5}{15}$ $\angle G \cong \angle G$

$30 = 30$

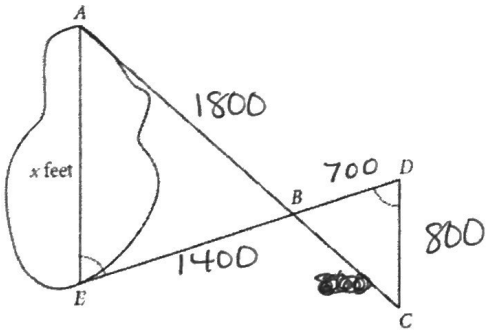
A. Similar but not congruent — SAS ~

B. Congruent but not similar — not $\cong$ b/c sides are not equal

C. Both similar and congruent

D. Neither similar nor congruent

14. ***SAT Prep:*** A summer camp counselor wants to find a length, x, in feet across a lake as represented in the sketch below. The lengths represented by AB, EB, BD, and CD on the sketch were determined to be 1800 feet, 1400 feet, 700 feet, and 800 feet respectively. Segments AC and DE intersect at B, and $\angle AEB$ and $\angle CDB$ have the same measure. What is the value of x?



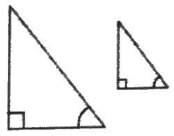
$\frac{700}{1400} = \frac{800}{x}$

$700x = 1{,}120{,}000$

$x = 1600$ ft

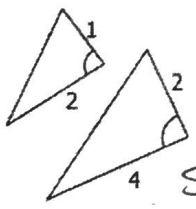
Identify which property will prove these triangles are similar (AA similarity, SAS similarity, SSS similarity)

15)



AA ~

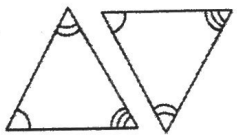
16)



$\frac{1}{2} = \frac{2}{4}$ ✓

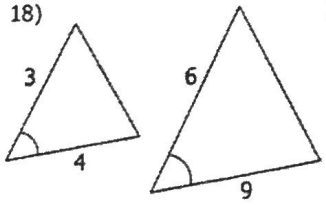
SAS ~

17)



AA ~

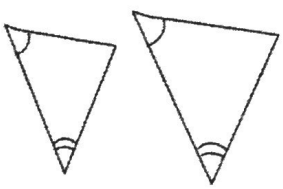
18)



$\frac{3}{6} = \frac{4}{9}$

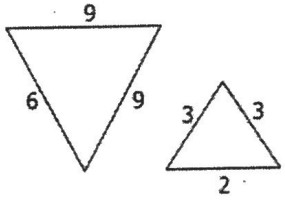
not similar

19)



AA ~

20)

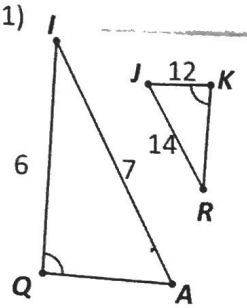


$\frac{2}{6} = \frac{3}{9} = \frac{3}{9}$ ✓

SSS ~

Write corresponding congruent angles and proportional sides. Then, identify which property will prove these triangles are similar (AA similarity, SAS similarity, SSS similarity). Write a similarity statement.

21)



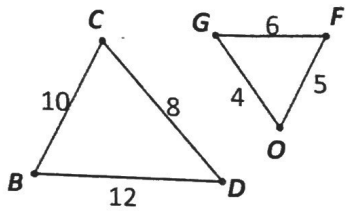
Corresponding parts: $\angle Q \cong \angle K$

not SAS ~

Are triangles similar? How? not similar

If so, state the similarity statement:

22)



Corresponding parts: S M L

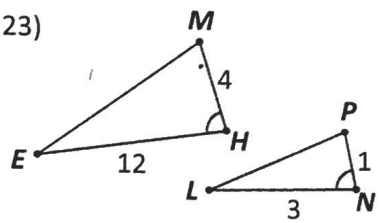
$\frac{8}{4} = \frac{10}{5} = \frac{12}{6} = 2$ ✓

Are triangles similar? How? SSS ~

If so, state the similarity statement:

$\triangle CBD \sim \triangle OGF$

23)



Corresponding parts: $\angle H \cong \angle N$

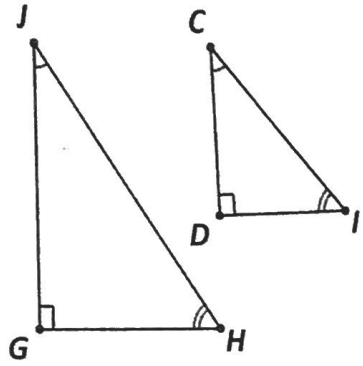
$\frac{4}{1} = \frac{12}{3} = 4$ ✓

Are triangles similar? How? SAS ~

If so, state the similarity statement:

$\triangle EHM \sim \triangle LNP$

24)



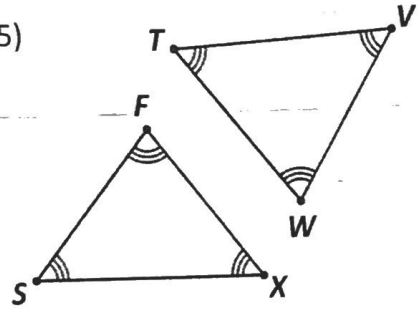
Corresponding parts: $\angle J \cong \angle C$
$\angle G \cong \angle D$
$\angle H \cong \angle I$

Are triangles similar? How? AA ~

If so, state the similarity statement:

$\triangle JGH \sim \triangle CDI$

25)



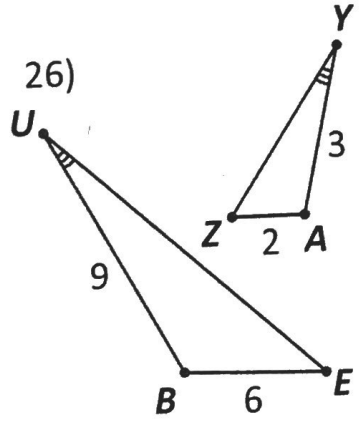
Corresponding parts: Equiangular

Are triangles similar? How? AA ~

If so, state the similarity statement:

$\triangle FSX \sim \triangle TVW$

26)



Corresponding parts:

not similar

Are triangles similar? How? not SAS ~

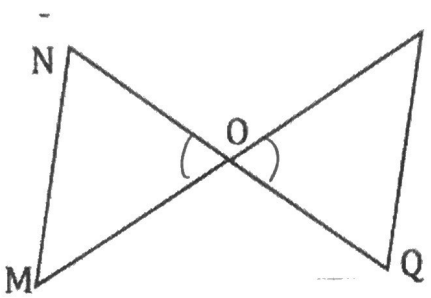
If so, state the similarity statement:

Lesson: Similar Triangle Proofs

Complete the given two-column proofs.

1. Given $\frac{NO}{QO} = \frac{MO}{PO}$

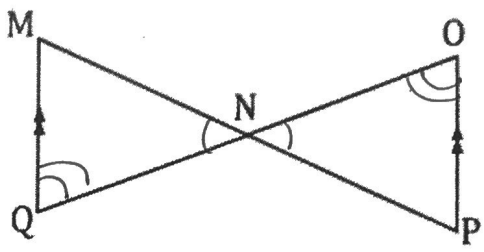
Prove: $\Delta MNO \sim \Delta PQO$



Statements	Reasons
1. $\frac{NO}{QO} = \frac{MO}{PO}$	1. Given
2. $\angle NOM \cong \angle QOP$	2. Vertical $\angle$s Conjecture
3. $\Delta MNO \sim \Delta PQO$	3. AA $\sim$

2. Given: $\overline{MQ} \parallel \overline{OP}$

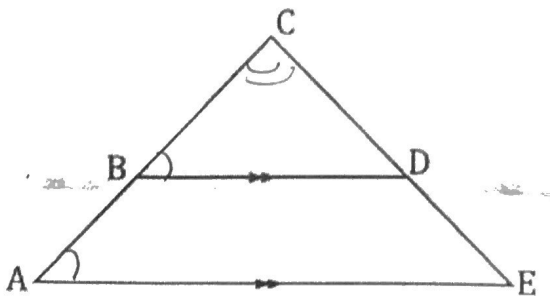
Prove: $\Delta MNQ \sim \Delta PON$



Statements	Reasons
1. $\overline{MQ} \parallel \overline{OP}$	1. given
2. $\angle Q \cong \angle O$	2. Alt. Int. $\angle$ Conjecture
3. $\angle QNM \cong \angle ONP$	3. Vertical $\angle$ Conjecture
4. $\Delta MNQ \sim \Delta PON$	4. AA $\sim$

3. Given: $\overline{AE} \parallel \overline{BD}$

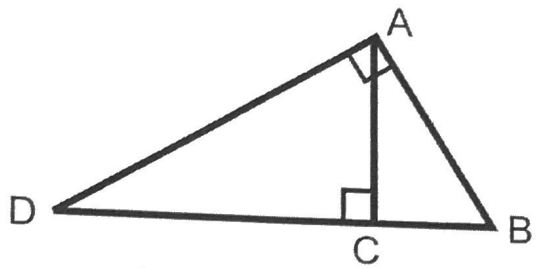
Prove: $\triangle ACE \sim \triangle BCD$

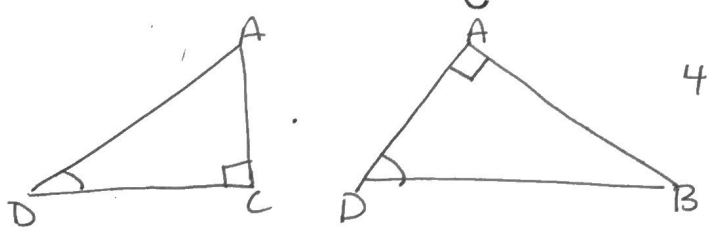


Statements	Reasons
1. $\overline{AE} \parallel \overline{BD}$	1. Given
2. $\angle CBD \cong \angle BAE$	2. Corresponding ∠ Conjecture
3. $\angle C \cong \angle C$	3. Reflexive Prop.
4. $\triangle ACE \sim \triangle BCD$	4. AA ~

4. Given $\triangle DAB$ and $\triangle DCA$ are right triangles

Prove: $\triangle DAB \sim \triangle DCA$





Statements	Reasons
1. $\triangle DAB$ and $\triangle DCA$ are right $\triangle$'s	1. given
2. $\angle DCA \cong \angle DAB$	2. Def. of right ∠'s
3. $\angle D \cong \angle D$	3. Reflexive Prop.
4. $\triangle DAB \sim \triangle DCA$	4. AA ~