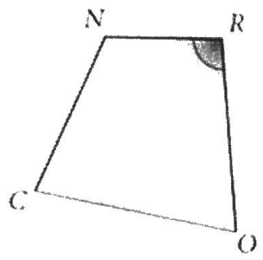
Unit 3 Notes - Introduction to Similar Triangles

Just as congruence introduced us to new notation, similarity will have its own set of notation.
If ΔCAT is congruent to ΔMEW, we write $\Delta CAT \cong \Delta MEW$. If two polygons are *similar* we use the symbol that sits above the equal sign in the congruent symbol: (~)
For example, the statement **CORN ~ PEAS** says that quadrilateral CORN is **similar** to quadrilateral PEAS. Just as in statements of congruence, the order of the letters tells you which segments and which angles in the two polygons correspond.



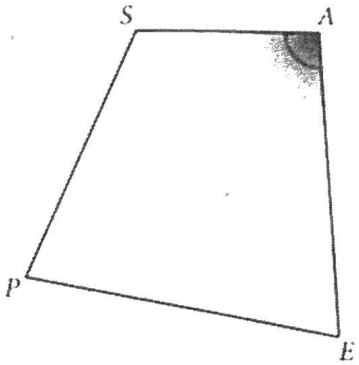
Corresponding angles are congruent:

$\angle C \cong \angle P$ $\angle R \cong \angle A$

$\angle O \cong \angle E$ $\angle N \cong \angle S$

Corresponding segments are proportional:

$\frac{CO}{PE} = \frac{OR}{EA}$



The ratio of the lengths of any two segments in one polygon is equal to the ratio of the corresponding two segments in the similar polygon. For example, $\frac{CO}{OR} = \frac{PE}{EA}$ or $\frac{NR}{CO} = \frac{SA}{PE}$

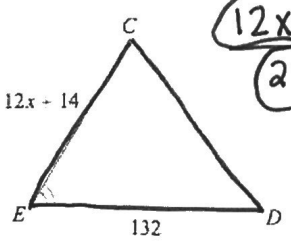
As we know, **Similar Figures** have the same shape, but not necessarily the same size.

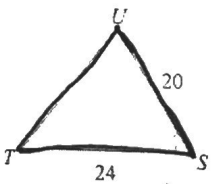
All corresponding angles are **equal** and corresponding sides are **proportional.** Proportionality is based on a scale factor which we will see later in transformation (dilations).

Learning Goal # 1: I can find the missing measurements of two similar figures.

Directions: Given the similarity statement, solve for the missing variable.

1. $\Delta CDE \sim \Delta UTS$





$\frac{12x+14}{20} = \frac{132}{24}$

$24(12x+14) = 20(132)$

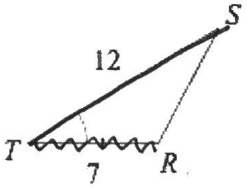
$288x + 336 = 2640$

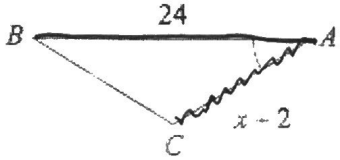
$-336 \quad -336$

$\frac{288x}{288} = \frac{2324}{288}$

$\boxed{x = 8}$

2. $\Delta SRT \sim \Delta BCA$





$\frac{x+2}{7} = \frac{24}{12}$

$12(x+2) = 7(24)$

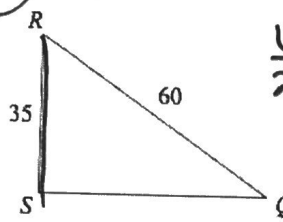
$12x + 24 = 168$

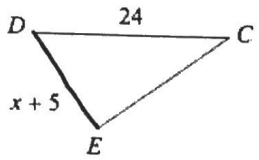
$-24 \quad -24$

$12x = 144$

$\boxed{x = 12}$

3. $\Delta RSQ \sim \Delta DEC$





$\frac{60}{24} = \frac{35}{x+5}$

$60(x+5) = 24(35)$

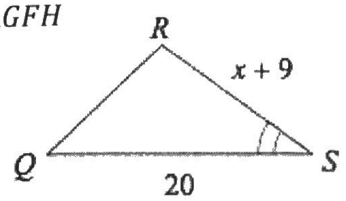
$60x + 300 = 840$

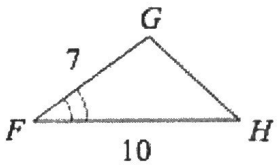
$-300 \quad -300$

$\frac{60x}{60} = \frac{540}{60}$

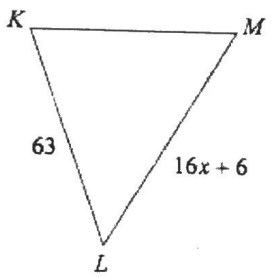
$x = 9$

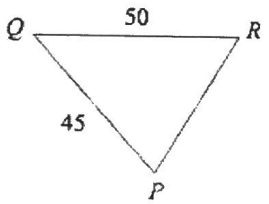
4. $\Delta RSQ \sim \Delta GFH$





5. $\Delta LKM \sim \Delta QPR$





6. $\Delta GHF \sim \Delta VWU$

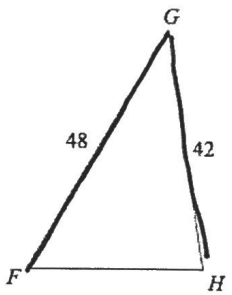
$\frac{48}{7} = \frac{48}{2x-16}$

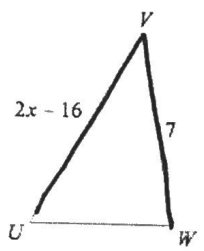
$42(2x-16) = 7(48)$

$84x - 672 = 336$

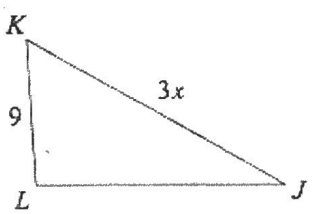
$\frac{84x}{84} = \frac{1008}{84}$

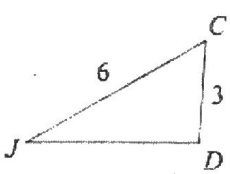
$x = 12$





7. $\Delta JKL \sim \Delta JCD$





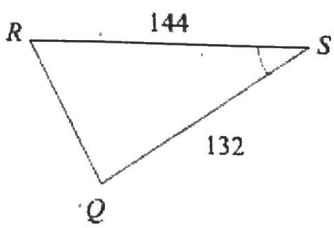
$\frac{24}{6} = \frac{12}{x}$

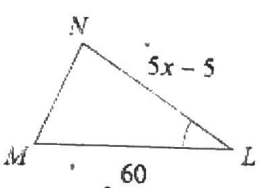
$24x = 6(12)$

$\frac{24x}{24} = \frac{72}{24}$

$x = 3$

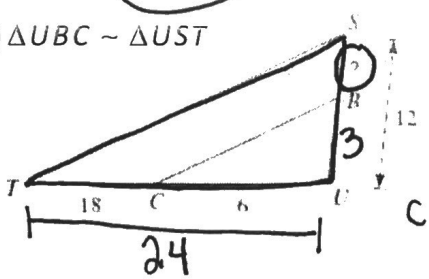
8. $\Delta QRS \sim \Delta NML$

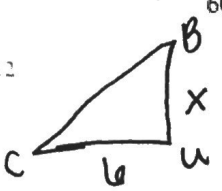




?=9

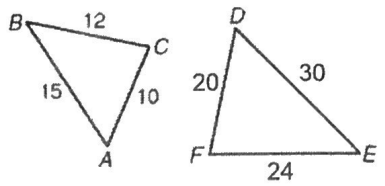
9. $\Delta UBC \sim \Delta UST$



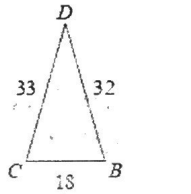


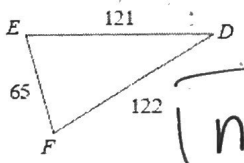
Similarity Statements

Complete the following graphic organizer:

Term/ Postulate	Definition/Explanation	Diagram
Side – Side - Side SSS~ must have	If three sides of a triangle are PROPORTIONAL to three corresponding sides of another triangle, then the triangles are similar.	 S m L $\frac{10}{20}$ ·.5 $\frac{12}{24}$ ·.5 $\frac{15}{30}$ = .5 Statement: Δ BCA ~ Δ EFD

Example: Which of the following example(s) can be shown triangles are similar by SSS~?



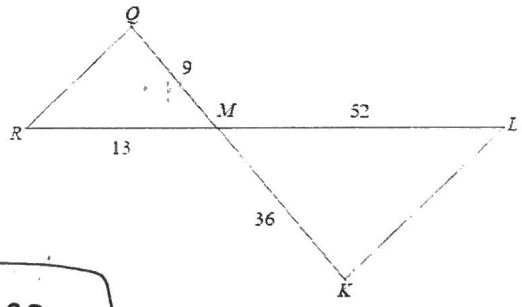


not SSS~

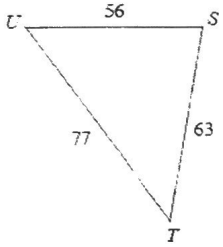
S: $\frac{18}{65}$ = .277

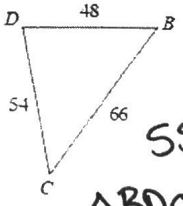
m: $\frac{32}{121}$ = .264

L: $\frac{33}{122}$ =



not SSS~
(no 3rd side)





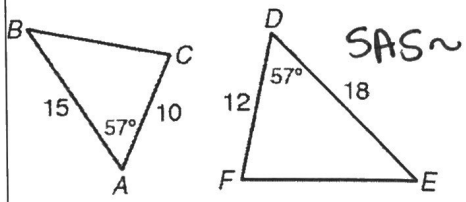
SSS~

Δ BDC ~ Δ UST

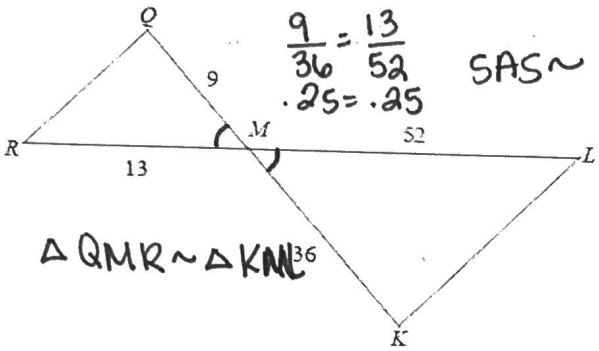
S: $\frac{48}{56}$ = .857

m: $\frac{54}{63}$ = .857

L: $\frac{66}{77}$ = .857

Side-Angle-Side **SAS~**	**If TWO sides of a triangle are PROPORTIONAL to the two corresponding sides of another triangle AND the INCLUDED angles are CONGRUENT, then the triangles are similar.**	 SAS~ $\frac{10}{12} = .85 \quad \frac{15}{18} = .85$ Statement: $\triangle ACB \sim \triangle DFE$

Example: Which of the following example(s) can be shown triangles are similar by SAS~?

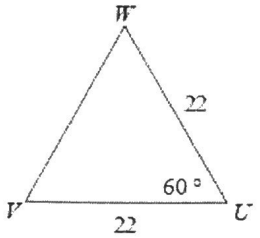


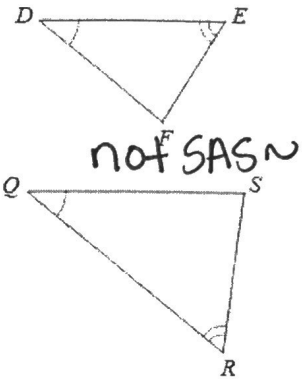
$\angle QMR \cong \angle KML$ (vertical)

$\frac{9}{36} = \frac{13}{52}$ SAS~

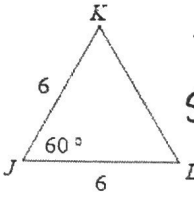
$.25 = .25$

$\triangle QMR \sim \triangle KML$





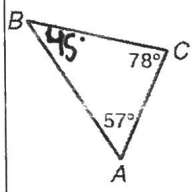
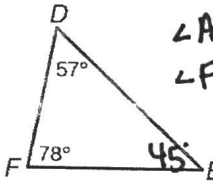
not SAS~



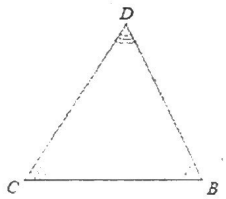
$\frac{6}{22} = \frac{6}{22}$

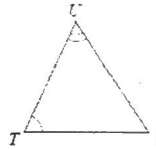
SAS~

$\triangle JKL \sim \triangle UVW$

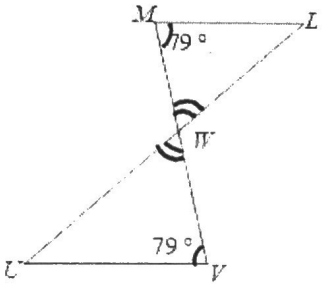
Angle- Angle **AA~**	**If two ANGLES of one triangle are CONGRUENT to two corresponding angles of another triangle, then the triangles are similar.**	  $\angle A \cong \angle D$ $\angle F \cong \angle C$ AA~ Statement: $\triangle ABC \sim \triangle FED$

Example: Which of the following example(s) can be shown triangles are similar by AA~?





not AA~

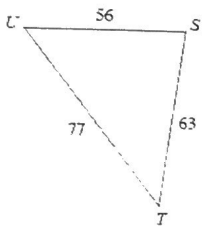


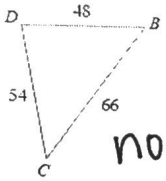
$\angle M \cong \angle V$

$\angle MWL \cong \angle VWU$

AA~

$\triangle MWL \sim \triangle VWU$





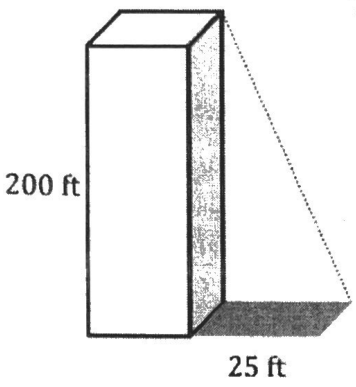
not AA~

Shadow Math

Finding the length of an unknown side:

* If you know that two objects are similar, you can set up a **proportion** to find the length of an unknown side.

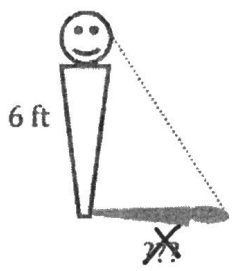
Example 1. A building has a shadow that is 25 feet long. A person 6 feet tall cast a similar shadow. How long is the person's shadow?



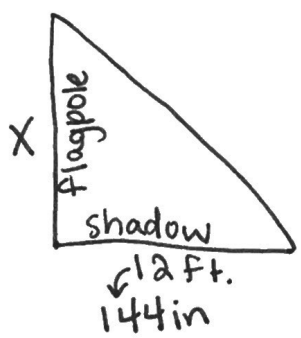
$\frac{200}{6} = \frac{25}{x}$

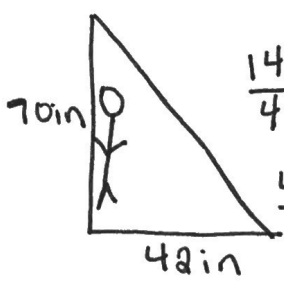
$200x = 150$

$x = 0.75$ ft.



Example 2: Suppose a person 5 feet 10 inches tall casts a shadow that is 3 feet 6 inches tall. At the same time of the day, a flagpole casts a shadow that is 12 feet long. To the nearest foot, how tall is the flagpole?





$12\text{ft}(12) = 144\text{in}$

$\frac{144}{42} = \frac{x}{70}$

$\frac{42x}{42} = \frac{10{,}080}{42}$

$x = \frac{240\text{in}}{12} = 20\text{ft}.$

$12\text{in} = 1\text{ft}$

$5\text{ft}(12) = 60\text{in}$

$+10\text{in}$

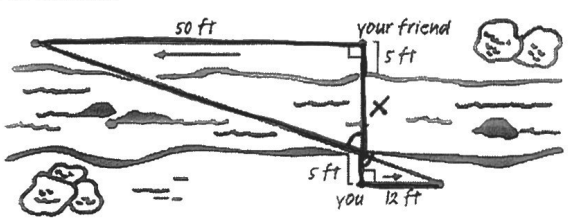
70in

$3\text{ft}(12) = 36\text{in}$

$+6\text{in}$

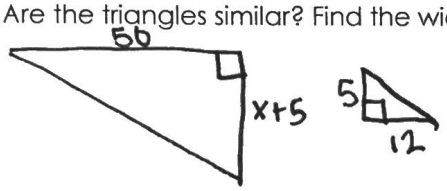
42in

Example 3: You and your friend are on opposite sides of the creek and are 5 feet from the creek bank. She walks 50 feet to the left on one side and you walk 12 feet to the right. Are the triangles similar? Find the width of the creek.



AA ~

90° & vertical



$\frac{50}{12} = \frac{5+x}{5}$

$12(5+x) = 5(50)$

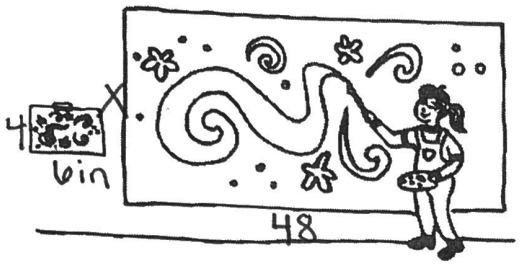
$60 + 12x = 250$

$12x = 190$

$x = 15.83$ ft

Notes

Example 4: A wall mural is being painted from a picture that is 6 inches long and 4 inches wide. The wall mural should be 48 inches long. The picture and wall mural are similar. How wide is the width of the mural? What is the scale factor of the picture to the mural?



$\frac{4}{x} = \frac{6}{48}$

$\frac{6x}{6} = \frac{192}{6}$

$x = 32$ in.

YOU DO- Shadow Math: Similar Triangles in the Real-world

1. A 6 ft tall circus tent casts a 10 ft long shadow. At the same time of day, a 9 ft tall elephant casts a shadow. How long is the elephant's shadow?

2. An giraffe that is 10.2 feet tall casts a shadow that is 6.8 feet long. At the same time of day, a girl cast a shadow that is 3.6 feet long. How tall is the girl?

3. A tree with a height of 4m casts a shadow 15 m long on the ground. How high is another tree that casts a shadow which is 20 m long?

4. A 15 ft tall statue standing next to an adult elephant casts an 18 ft shadow. If the adult elephant is 10 ft tall, then how long is its shadow?