

Properties of Quadrilaterals

A **parallelogram** is a type of quadrilateral that has **two pairs of opposite sides that are parallel**. Parallelograms are denoted by the symbol \square . If a quadrilateral has two pairs of parallel, opposite sides, then it can be classified as a parallelogram.

There are 5 theorems associated with PARALLELOGRAMS:

- Opposite sides are congruent

$$\overline{KL} \cong \overline{NM} \text{ and } \overline{KN} \cong \overline{ML}$$

- Opposite angles are congruent

$$\angle K \cong \angle M, \angle N \cong \angle L$$

- Consecutive angles are supplementary

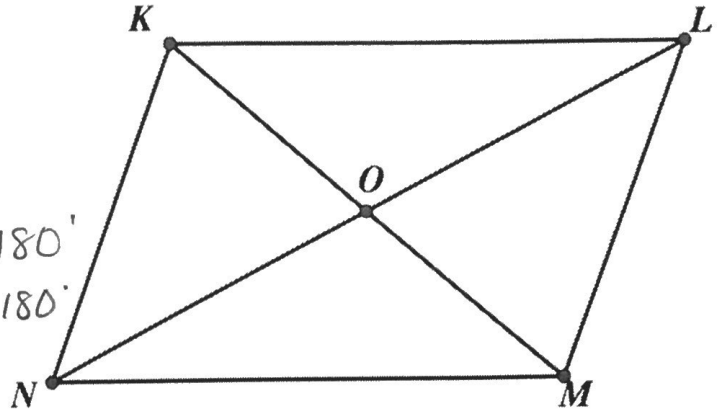
$$\begin{aligned} \angle N + \angle K &= 180^\circ & \angle L + \angle M &= 180^\circ \\ \angle K + \angle L &= 180^\circ & \angle M + \angle N &= 180^\circ \end{aligned}$$

- Diagonals bisect each other

$$\begin{aligned} \overline{KO} &\cong \overline{OM} \\ \overline{NO} &\cong \overline{OL} \end{aligned}$$

- Diagonals form two congruent triangles

$$\triangle KON \cong \triangle MOL, \triangle KLO \cong \triangle MNO$$



Parallelograms can be broken down into three more specific types of quadrilaterals with the same properties as parallelograms. The three specific types also have some of their own properties.

Rectangles



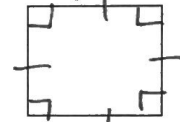
- All properties of parallelograms
- Diagonals are congruent
- Four right angles

Rhombus



- All properties of parallelograms
- Diagonals are perpendicular
- Diagonals bisect each other
- Four sides are congruent

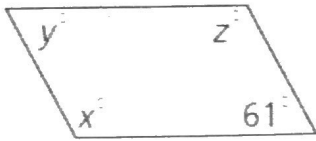
Square



- All properties of parallelograms
- Four right angles
- Four congruent sides
- Diagonals are congruent, perpendicular, and bisect each other

Applying Properties of Quadrilaterals

1. Solve for x, y, and z.



$$180 - 61 = 119^\circ$$

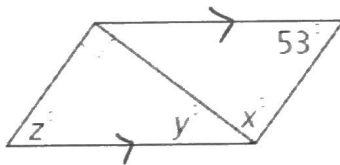
$$x^\circ = z^\circ = 119^\circ$$

$$y = 61^\circ$$

Relationship: parallelogram

- opposite \angle 's are \cong
- Consecutive \angle 's are supplementary

2. Solve for x, y, and z.



$$z = 53^\circ$$

$$x^\circ = 90^\circ$$

$$y^\circ = 180 - 143 = 37^\circ$$

Relationship: parallelogram

- parallel lines cut by a trans. create a Alt. Int. \angle 's \cong .
- opposite \angle 's are \cong

3. In parallelogram ABCD, AB = 17.5, DE = 18, and $m\angle BCD = 110^\circ$. Point E represents the intersection of the diagonals. Draw a picture of parallelogram ABCD and answer the following questions:

a. $BD = 18 + 18 = 36$

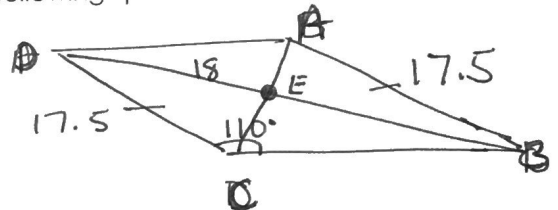
b. $CD = 17.5$

c. $BE = 18$

d. $m\angle ABC = 180 - 110 = 70^\circ$

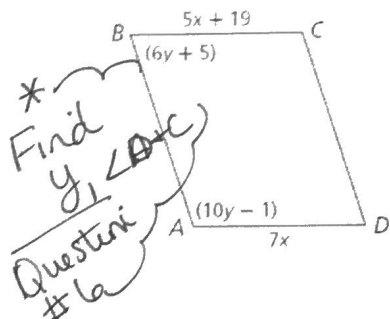
e. $m\angle ADC = 70^\circ$

f. $m\angle DAB = 110^\circ$



4. Find the value of x. Then find the length of BC.

Relationship: Consecutive \angle 's are Suppl.



$$5x + 19 = 7x$$

$$19 = 2x$$

$$9.5 = x$$

• opposite sides are \cong

$$BC = 5(9.5) + 19$$

$$66.5$$

$$6y + 5 + 10y - 1 = 180$$

$$16y + 4 = 180$$

$$16y = 176$$

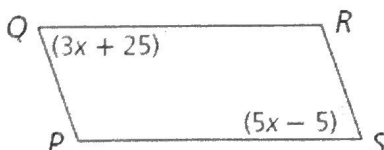
$$y = 11$$

$$\angle C = \angle A = \frac{10(11) - 1}{2} = 109^\circ$$

$$\angle D = 180 - 71 = 109^\circ$$

5. Find the value of x. Then find Angle Q.

Relationship: opposite \angle 's are \cong



$$3x + 25 = 5x - 5$$

$$+5 \quad -3x$$

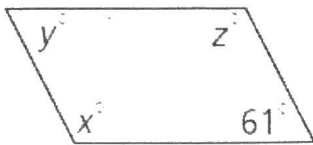
$$30 = 2x$$

$$15 = x$$

$$\angle Q = 3(15) + 25 = 70^\circ$$

Applying Properties of Quadrilaterals

1. Solve for x, y, and z.



$$180 - 61 = 119^\circ$$

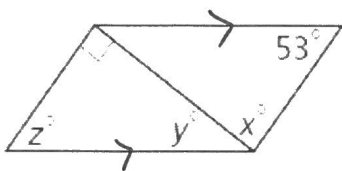
$$x^\circ = z^\circ = 119^\circ$$

$$y = 61^\circ$$

Relationship: parallelogram

- opposite \angle 's are \cong
- consecutive \angle 's are supplementary

2. Solve for x, y, and z.



$$z = 53^\circ$$

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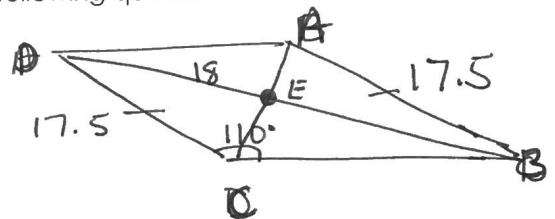
Relationship: parallelogram

- parallel lines cut by a trans. create a Alt. Int. \angle 's \cong .
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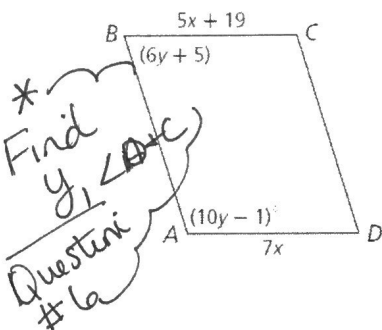
- a. $BD = 18 + 18 = 36$
- c. $BE = 18$
- e. $m\angle ADC = 70^\circ$

- b. $CD = 17.5$
- d. $m\angle ABC = 180 - 110 = 70^\circ$
- f. $m\angle DAB = 110^\circ$



4. Find the value of x. Then find the length of BC.

Relationship: Consecutive \angle 's are suppl.



$$5x + 19 = 7x$$

$$19 = 2x$$

$$9.5 = x$$

• opposite sides are \cong

$$BC = 5(9.5) + 19$$

$$66.5$$

$$6y + 5 + 10y - 1 = 180$$

$$16y + 4 = 180$$

$$16y = 176$$

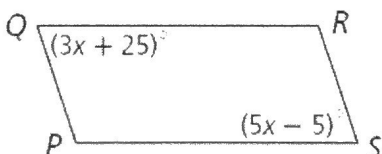
$$y = 11$$

$$\angle C = \angle A = (10(11) - 1) = 109^\circ$$

$$\angle D = 71^\circ$$

5. Find the value of x. Then find Angle Q.

Relationship: opposite \angle 's are \cong



$$3x + 25 = 5x - 5$$

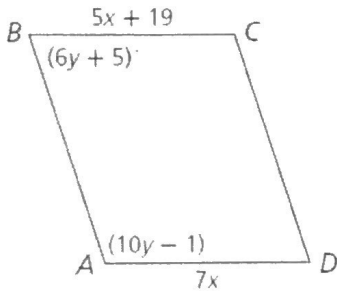
$$+5 \quad -3x$$

$$30 = 2x$$

$$15 = x$$

$$\angle Q = 3(15) + 25 = 70^\circ$$

6. Find the value of y . Then find the measure of Angle C and D.



Relationship: Consecutive x 's are
Suppl., opposite ^{angles}
are $\underline{=}$

$$6y + 5 + 10y - 1 = 180$$

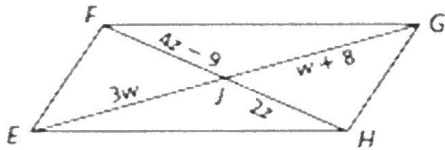
$$16y + 4 = 180$$

$$y = 11$$

$$\angle A = \angle C = (10(11) - 1) = 109^\circ$$

$$\angle B = \angle D = 6(11) + 5 = 71^\circ$$

7. EFGH is a parallelogram. Find w and z .



Relationship: Diagonals bisect
each other

$$4z - 9 = 2z$$

$$-9 = -2z$$

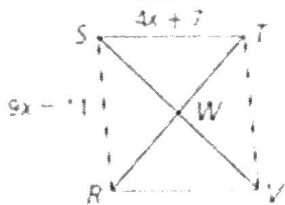
$$\boxed{4.5 = z}$$

$$3w = w + 8$$

$$2w = 8$$

$$\boxed{w = 4}$$

8. RSTV is a rhombus. Find the length of TV.



Relationship: all sides are $\underline{=}$

$$4x + 7 = 9x - 11$$

$$+11 \quad -4x$$

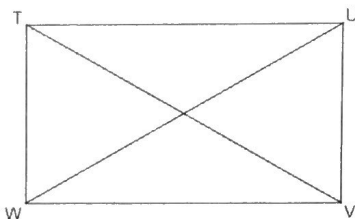
$$\hline 18 = 5x$$

$$x = 3.6$$

$$\overline{ST} \cong \overline{TV} = 4(3.6) + 7$$

$$\boxed{21.4}$$

9. In rectangle TUVW below, it is known that $TV = 19 - 2x$ and $WU = 10 + x$. Find the value of x .



Relationship: Diagonals are $\underline{=}$

$$19 - 2x = 10 + x$$

$$-10$$

$$+2x$$

$$\hline 9 = 3x$$

$$\boxed{3 = x}$$