Learning Task: Deriving the Equation of a Circle

The Return of Pythagoras

Below is a circle of radius 3, drawn on a coordinate plane centered at the origin. Four points are marked on the circle. Each member of your group should choose one of the four points and draw a right triangle that connects the chosen point to the origin. Use the given coordinates to label the legs of your right triangle. Then apply the Pythagorean Theorem to determine the hypotenuse of the triangle.

Calculation for your right triangle

$leg^2 + leg^2 = hypotenuse^2$

$\cancel{(-\sqrt{4.5})^2} + \cancel{(\sqrt{4.5})^2} = r^2$

$4.5 + 4.5 = r^2$

$\sqrt{9} = \sqrt{r^2}$

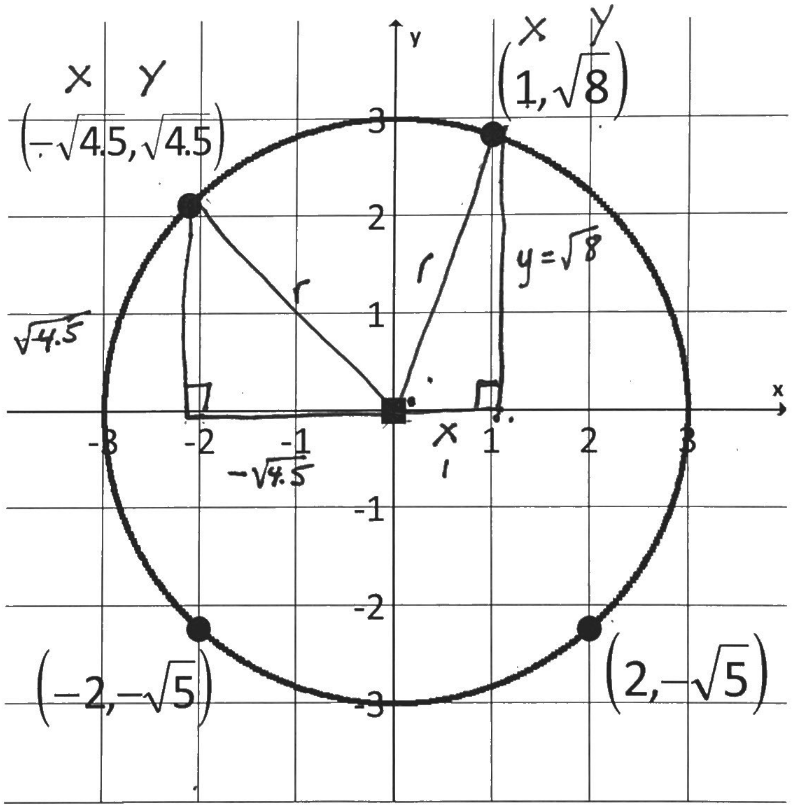
$\boxed{3 = r}$

$(1)^2 + \cancel{(\sqrt{8})^2} = r^2$

$1 + 8 = r^2$

$\sqrt{9} = \sqrt{r^2}$

$\boxed{3 = r}$

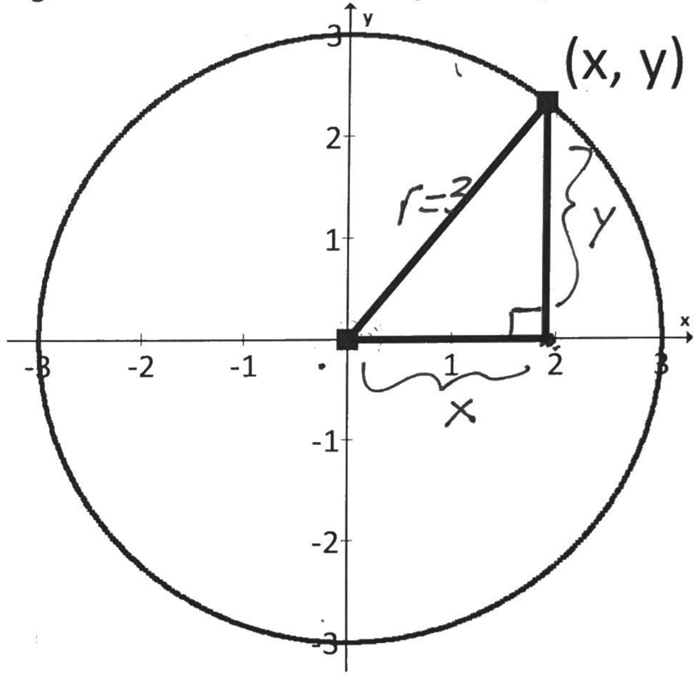


1. Share your answers with your group. What do you notice?

2. Now that you have seen how the Pythagorean Theorem relates to the radius of a circle, you will develop that relationship in a more general sense. An *arbitrary point* has been placed on the circle of radius 3. A right triangle has been drawn in for you as well. **Label the triangle's legs and hypotenuse, and then write the Pythagorean Theorem that models your triangle.**

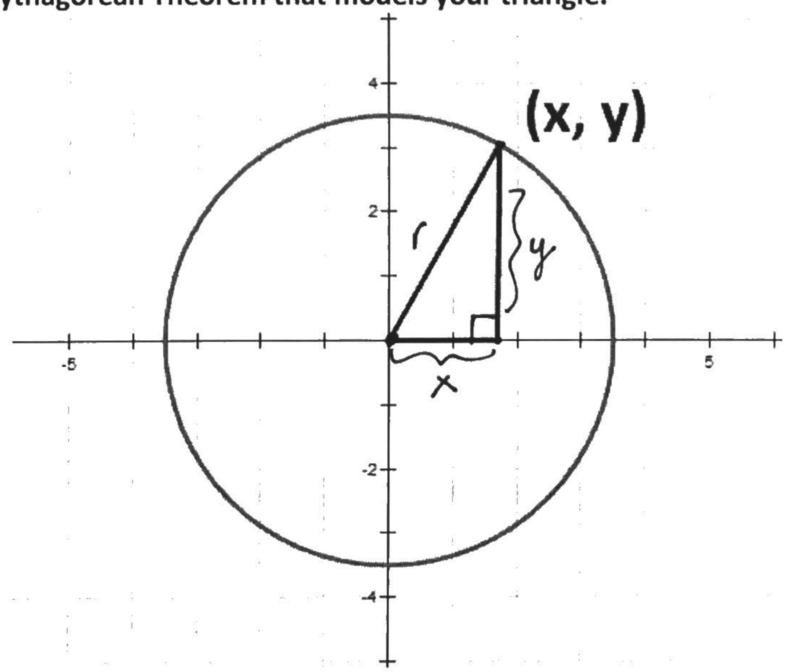
$(x)^2 + (y)^2 = (3)^2$

$\boxed{x^2 + y^2 = 9}$



3. Finally, try to move to a more general circle. This time, not only is the point arbitrary, but the radius is as well. Call the radius *r*. **Similar to problem #2, label the right triangle's legs and hypotenuse and then write the Pythagorean Theorem that models your triangle.**

$x^2 + y^2 = r^2$



Standard form Circle equation with Center at the Origin:

$x^2 + y^2 = r^2$

Circles, Transformed

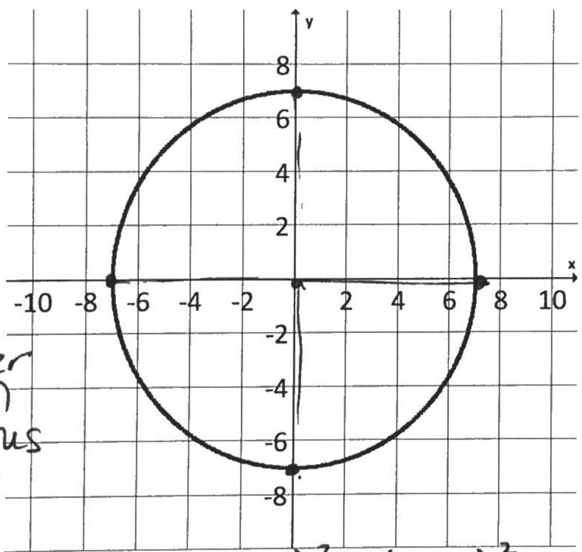
Standard Form of a Circle

$(x-h)^2 + (y-k)^2 = r^2$

where (h,k) is Center and r is the radius

Using a Circle's Equation

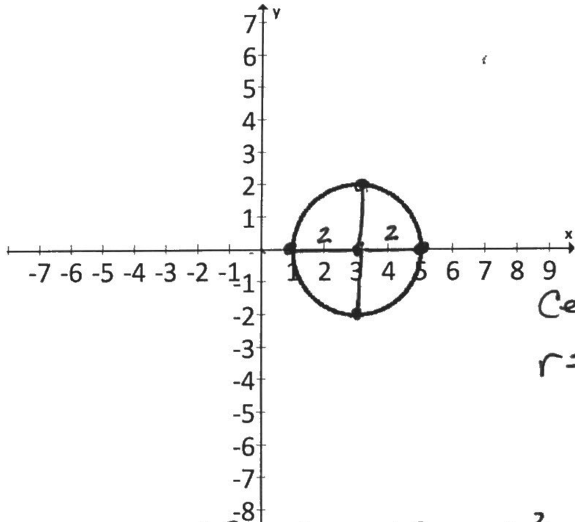
1. Write the equation of each circle below.



1. Center (0,0)
2. radius 7

$(x-0)^2 + (y-0)^2 = 7^2$

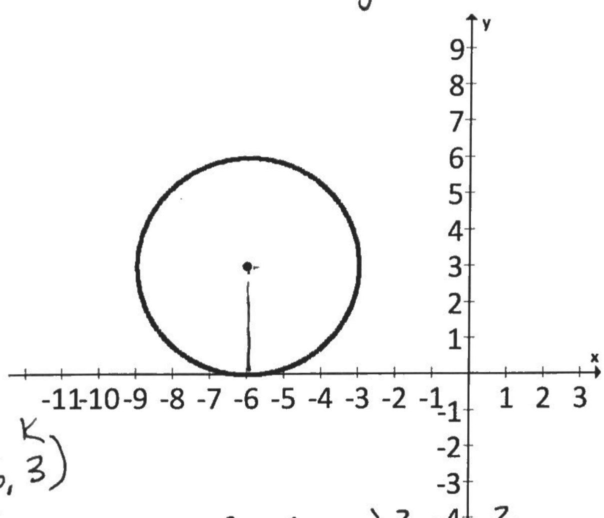
Equation: $x^2 + y^2 = 49$



h k
Center (3,0)
r = 2

$(x-3)^2 + (y-0)^2 = (2)^2$

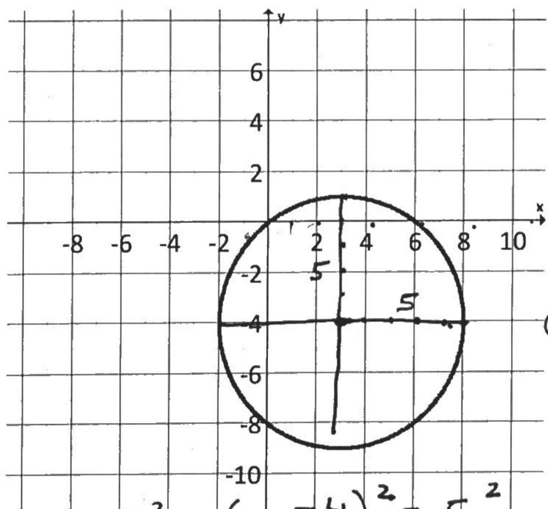
Equation: $(x-3)^2 + y^2 = 4$



h k
C (-6, 3)
r = 3

$(x - {}^-6)^2 + (y-3)^2 = 3^2$

Equation: $(x+6)^2 + (y-3)^2 = 9$



h k
C (3, -4)
r = 5

$(x-3)^2 + (y - {}^-4)^2 = 5^2$

Equation: $(x-3)^2 + (y+4)^2 = 25$

$(x-h)^2+(y-k)^2=r^2 \quad C(h,k) \quad \text{radius}=\sqrt{\ } = r$

2. Given the equation of the circle, identify the radius and center for each circle. Leave answers in simplest radical form.

$x^2+(y-8)^2=100$	$(x+1)^2+(y-4)^2=1$	$(x-10)^2+(y+3)^2=50$	$(x+2)^2+(y-2)^2=45$
Radius: $\sqrt{100}=10$	Radius: $\sqrt{1}=1$	Radius: $\sqrt{50}=5\sqrt{2}$	Radius: $\sqrt{45}=3\sqrt{5}$
Center: $(0, 8)$	Center: $(-1, 4)$	Center: $(10, -3)$	Center: $(-2, 2)$

$\sqrt{50}$

$\boxed{2} \cdot \cancel{25}$

$5 \cdot 5$

$\sqrt{45}$

$\cancel{9} \cdot \boxed{5}$

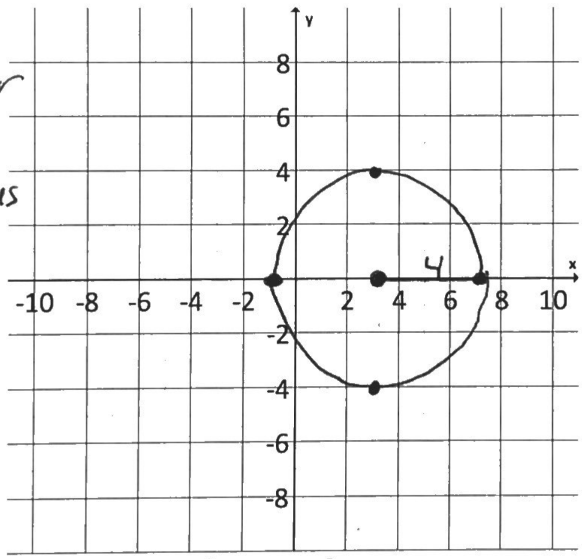
$3 \cdot 3$

3. Use the equation to graph each circle.

$(x-3)^2+y^2=16$

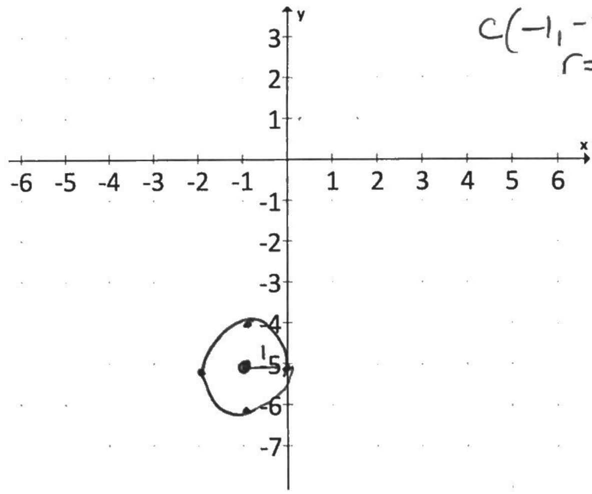
1. Center $(3, 0)$

2. Radius $\sqrt{16}=4$



$(x+1)^2+(y+5)^2=1$

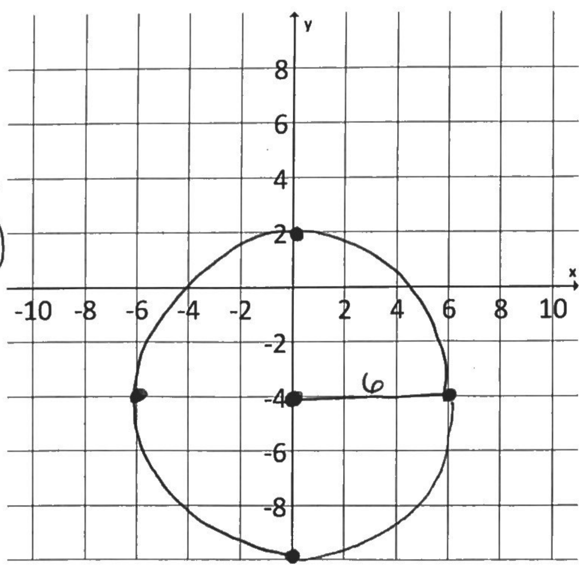
$C(-1,-5)$
$r=1$



$x^2+(y+4)^2=36$

Center $(0, -4)$

$r=\sqrt{36}$
6



$(x-5)^2+(y-5)^2=25$

$C(5, 5)$
$r=5$