NOTES-Proving Rhombus and Squares on a Coordinate Plane

PARALLELOGRAMS ON THE COORDINATE PLANE

Objectives:

- Show that a quadrilateral is a parallelogram on the coordinate plane
- Identify and verify parallelograms

DISTANCE FORMULA: $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

MIDPOINT FORMULA: $(x_m, y_m) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

SLOPE FORMULA: $m = \frac{y_2 - y_1}{x_2 - x_1}$

FORMULAS & THE COORDINATE PLANE	
FORMULA	WHEN TO USE IT
Distance Formula: $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$	To determine whether... • Sides are congruent • Diagonals are congruent
Midpoint Formula: $(x_m, y_m) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$	To determine... • The coordinates of a midpoint of a side • Whether diagonals bisect each other
Slope Formula: $m = \frac{y_2 - y_1}{x_2 - x_1}$	To determine whether... • Opposite sides are parallel • Diagonals are perpendicular • Sides are perpendicular

QUADRILATERAL	PROVE:
RHOMBUS	First prove it's a parallelogram, and then prove... • Two consecutive sides are congruent • The diagonals are perpendicular slope OR... • All four sides are congruent distance
SQUARE	• It's a rectangle and a rhombus (see above)

1. all 4 angles are Right
 $\perp$ lines
2. Diagonals are $\cong$ Diagonals $\perp$

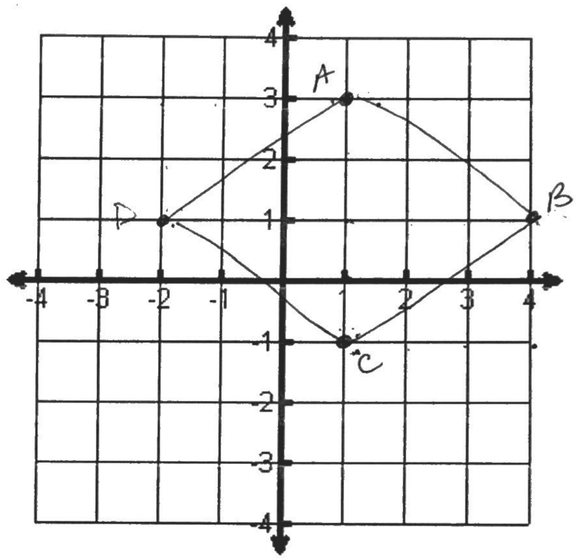
Geometry — Unit 7—Coordinate Geometry — Notes

Proving a Quadrilateral is a Rhombus

Method: Prove that all four sides are congruent.

Example 1: Plot and label each point. A(1, 3), B(4, 1), C(1, -1), and D(-2, 1)

Prove it!



Find the **length** of each side ~~to the nearest tenth~~.

AB = $\sqrt{13}$ $\quad d=\sqrt{(4-1)^2+(1-3)^2}$ $\quad \sqrt{9+4}$

BC = $\sqrt{13}$ $\quad d=\sqrt{(1-4)^2+(-1-1)^2}$ $\quad \sqrt{9+4}$

DC = $\sqrt{13}$ $\quad d=\sqrt{(-2-1)^2+(1+1)^2}$ $\quad \sqrt{9+4}$

DA = $\sqrt{13}$ $\quad d=\sqrt{(-2-1)^2+(1-3)^2}$ $\quad \sqrt{9+4}$

- What conclusions can you make? (Hint: are any sides the same length)

all 4 sides are $\cong$. $\overline{AB} \cong \overline{BC} \cong \overline{DC} \cong \overline{DA}$

Find the **slope** of each side.

Slope of AB = $\frac{-2}{3}$ $\qquad \overline{AB} \parallel \overline{DC}$ $\qquad \frac{2}{3} \cdot \frac{-2}{3} = \frac{-4}{9} \neq -1$

don't have $\perp$ lines

Slope of DC = $\frac{2}{-3}$

Slope of BC = $\frac{-2}{-3} = \frac{2}{3}$

$\overline{BC} \parallel \overline{AD}$

Slope of AD = $\frac{2}{3}$ $\qquad$ opposite sides are $\not\parallel$ parallel

- What conclusions can you make? (Hint: are any sides parallel? Perpendicular ?)

Based on my answers above, I have proven this shape to be a Rhombus

because...

1) opposite sides are parallel (parallelogram)
2) all 4 sides are $\cong$ (Rhombus)

Proving that a Quadrilateral is a Square

Method: First, prove the quadrilateral is a rhombus by showing all four sides is congruent; then prove the quadrilateral is a rectangle by showing the diagonals is congruent.

Example 2: Plot and label each point. A(-5, 6), B(3, 7), C(4, -1), and D(-4, -2)

Prove it!

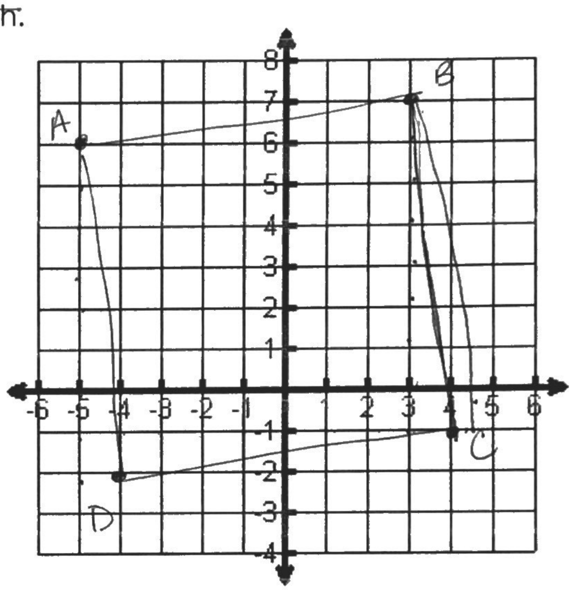
Find the **length (distance)** of each side ~~to the nearest tenth.~~

AB = $\sqrt{65}$ $d=\sqrt{(3+5)^2+(7-6)^2}$ $\sqrt{64+1}$

BC = $\sqrt{65}$ $d=\sqrt{(4-3)^2+(-1-7)^2}$ $\sqrt{1+64}$

DC = $\sqrt{65}$ $d=\sqrt{(-4-4)^2+(-2+1)^2}$ $\sqrt{64+1}$

DA = $\sqrt{65}$ $d=\sqrt{(-4+5)^2+(-2-6)^2}$ $\sqrt{7+64}$



- What conclusions can you make? (Hint: are any sides the same length?)

all sides are $\cong$ $\overline{AB} \cong \overline{BC} \cong \overline{DC} \cong \overline{DA}$

Find the **slope** of each side.

Slope of AB = $\frac{1}{8}$

$\overline{AB} \parallel \overline{DC}$

Slope of DC = $1/8$

Slope of BC = $\frac{-8}{1} = -8$

$\overline{BC} \parallel \overline{AD}$

Slope of AD = $\frac{-8}{1} = -8$

opposite sides are parallel

$\frac{1}{8} \cdot \frac{-8}{1} = \frac{-8}{8} = -1$

side are $\perp$

$\overline{AB} \perp \overline{BC}$ $\overline{CD} \perp \overline{DA}$

$\overline{BC} \perp \overline{CD}$ $\overline{AD} \perp \overline{AB}$

all angles are rt. $\angle$'s

- What conclusions can you make? (Hint: are any sides parallel? Perpendicular ?)

Based on my answers above, I have proven this shape to be a Square

because...

1.) Parallelogram – opposites sides are $\parallel$

2) Rectangle – all angles are 90°

3) Rhombus – all sides are $\cong$.

Geometry Unit 7—Coordinate Geometry Notes

GUIDED PRACTICE

1.)

Prove that a quadrilateral with the vertices A(-1,3), B(3,6), C(8,6) and D(4,3) is a rhombus.

1) Prove Parallelogram

$m\overline{AB} = \frac{3}{4}$ $m\overline{BC} = \frac{0}{5} = 0$

$m\overline{CD} = \frac{-3}{-4} = \frac{3}{4}$ $m\overline{DA} = \frac{0}{5} = 0$

* $\overline{AB} \parallel \overline{CD}$ $\overline{BC} \parallel \overline{DA}$

opposite sides are parallel

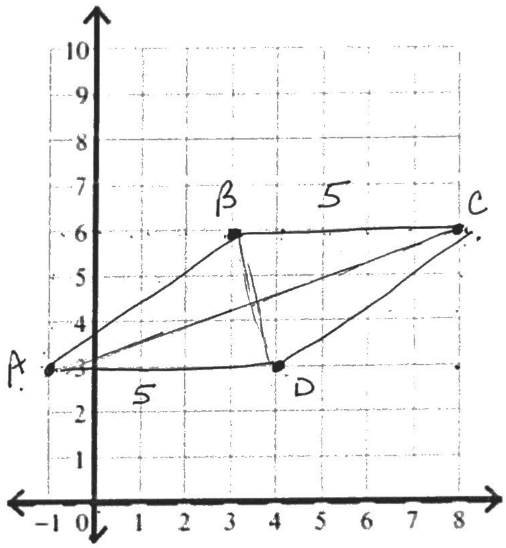
2) Prove Rhombus

$\overline{AC} \perp \overline{BD}$

$m\overline{AC} = \frac{3}{9} = \frac{1}{3}$

$m\overline{BD} = \frac{3}{-1} = -3$

$\frac{1}{3} \cdot \frac{-3}{1} = \frac{-3}{3} = -1$ ✓



2.)

Prove that the quadrilateral with vertices A(-1,0), B(3,3), C(6,-1) and D(2,-4) is a square.

1. Prove Parallelogram

$m\overline{AB} = \frac{3}{4}$ $m\overline{BC} = \frac{-4}{3}$

$m\overline{CD} = \frac{-3}{-4} = \frac{3}{4}$ $m\overline{DA} = \frac{4}{-3}$

$\overline{AB} \parallel \overline{CD}$ $\overline{BC} \parallel \overline{DA}$ opp. sides are $\parallel$.

2. Prove Rhombus

$m AC = \frac{-1}{7}$ $m\overline{BD} = \frac{-7}{-1} = \frac{7}{1}$

$\overline{AC} \perp \overline{BD}$

$\frac{-1}{7} \cdot \frac{7}{1} = \frac{-7}{7} = -1$ ✓

Diagonals are $\perp$

3. Prove Rectangle

$\overline{AC} \cong \overline{BD}$

$AC = \sqrt{(6+1)^2 + (-1-0)^2} = \sqrt{49+1} = \sqrt{50}$ ✓

$BD = \sqrt{(2-3)^2 + (-4-3)^2} = \sqrt{1+49} = \sqrt{50}$

Diagonals are $\cong$