Learning Task: Independent vs. Dependent

Events are **independent** when the outcome of one event **does not influence the outcome** of a second event. **When the outcome of one event affects the outcome** of a second event, the events are **dependent**.

To find the probability of two independent events both occurring, **multiply** the probability of the first event by the probability of the second event.

$$P(A \text{ and } B) = P(A) \bullet P(B)$$

To find the probability of two dependent events both occurring, **multiply** the probability of A and the probability of B after A occurs.

$$P(A \text{ and } B) = P(A) \bullet P(B \text{ after A occurs})$$

Determine if the following events are independent or dependent.

- You roll a die and flip a coin. Ind.
- You select one marble, do not replace it, then select another marble.  Dep.
- Selecting a marble and then choosing a second marble with replacing the first marble Ind.
- Rolling a number cube and spinning a spinner Dep.

Guided Practice

1. A bag contains 2 yellow, 12 red, and 6 green marbles. = 20 total
   a. What is the probability of selecting a red marble, replacing it, then selecting another red marble? P(red, red)

$$\frac{12}{20} \cdot \frac{12}{20} = \frac{9}{25} = .36$$

   b. What is the probability of selecting a red marble, not replacing it, then selecting another red marble? P(red, red)

$$\frac{12}{20} \cdot \frac{11}{19} = \frac{33}{95} = .35$$

   c. What is the probability of selecting 1 yellow marble, not replacing it, then selecting a green marble? P(Y, G)

$$\frac{2}{20} \cdot \frac{6}{19} = \frac{3}{95} = .03$$

2. There are 7 girls and 3 boys in a class. **Two students** are to be randomly chosen for a special project. Total: 10 — without replacement!

   a. What is the probability both students will be girls? P(girl, then different girl)

   $\frac{7}{10} \cdot \frac{6}{9} = .47$

   b. What is the probability both students are boys? P(boy, then different boy)

   $\frac{3}{10} \cdot \frac{2}{9} = .07$

   c. What is the probability of selecting a boy and a girl? P(boy, then girl)

   $\frac{3}{10} \cdot \frac{7}{9} = .23$

3. A card is drawn from a deck of cards. after you draw a card, you don't replace it:

   a. independent or dependent? Dependent

   b. P(club, ace) $\frac{13}{52} \cdot \frac{4}{51} = .02$

   c. P(king and then king) $\frac{4}{52} \cdot \frac{3}{51} = .005$

   52: 13 Hearts, 13 Diamonds > Red; 13 Spades, 13 Clubs > Black
   Face(J, Q, K) Letter(J, Q, K, Ace) Number 2...10

4. There are 3 blue, 4 orange, and 6 red marbles in a bag. What is the probability of picking an orange marble out of the bag? Total: 13

   $P(O) = 4/13 = .31$

5. A bag contains 6 red balls and 5 green balls. You randomly draw one ball, replace it, and randomly draw a second ball. What is the probability that the first ball is green and the second ball is red? P(G, R) Total 11

   $\frac{5}{11} \cdot \frac{6}{11} = .25$

6. An aquarium contains 6 male goldfish and 4 female goldfish. You randomly select a fish from the tank, do not replace it, and then randomly select a second fish. What is the probability that both fish are male? Total: 10

   P(M, M) $\frac{6}{10} \cdot \frac{5}{9} = .33$

7. A die is rolled {1, 2, 3, 4, 5, 6} and a spinner like the one at the right is spun.

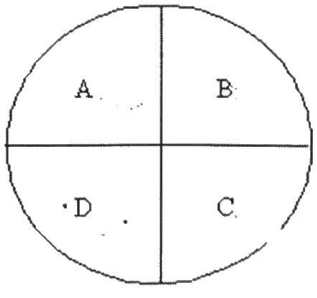
   a. Are the events independent or dependent?

   b. *P* (4 and A) $\frac{1}{6} \cdot \frac{1}{4} = \frac{1}{24} = .04$

   c. *P* (an even number and C) $\frac{3}{6} \cdot \frac{1}{4} = \frac{3}{24} = \frac{1}{8} = .125$

   d. *P* (2 or 5 and B or D)

   $\frac{2}{6} \cdot \frac{2}{4} = \frac{4}{24} = \frac{1}{6} = .17$



Learning Task: The Land of Independence

Independent events are defined by the equation $P(A \cap B) = P(A) \cdot P(B)$.
In other words, understand that if two events A and B are independent, the probability of A and B occurring together is the product of their probabilities.

1. Based on the definition of independence, determine if each set of events below are independent.

a. $P(A) = 0.45, P(B) = .30, P(A \cap B) = 0.75$

$.75 = (.45)(.30)$
$.75 \neq .135$ not indep.

b. $P(A) = 0.12, P(B) = .56, P(A \cap B) = 0.0672$

$.0672 = (.12)(.56)$ ✓ indep.
$.0672$

c. $P(A) = \frac{4}{5}, P(B) = \frac{3}{8}, P(A \cap B) = \frac{7}{40}$

$\frac{7}{40} \neq (\frac{4}{5})(\frac{3}{8})$ not indep.
$\frac{3}{10}$

d. $P(A) = \frac{7}{9}, P(B) = \frac{3}{4}, P(A \cap B) = \frac{7}{12}$

$\frac{7}{12} = (\frac{7}{9})(\frac{3}{4})$ ✓ independent
$\frac{7}{12}$

2. Determine the missing values so that the events *A* and *B* will be independent.

a. $P(A) = 0.55, P(B) =$ .25 $, P(A \cap B) = 0.1375$

$P(A \cap B) = P(A) \cdot P(B)$
$\frac{.1375}{.55} = \frac{(.55)(B)}{.55}$

b. $P(A) =$ 10/21 $, P(B) = \frac{3}{10}, P(A \cap B) = \frac{1}{7}$

$\frac{1}{7} = (A)(\frac{3}{10})$
$\frac{3}{10}$ $\frac{3}{10}$
$\frac{10}{21} = A$

3. Much of the data collected every 10 years for the Census is available to the public. This data includes a variety of information about the American population at large such as age, income, family background, education history and place of birth. Your job will be to use your knowledge of conditional probability and independence to make conclusions about the American populace.

   Has the gender gap closed in the world today? Are men and women able to earn the same amount of money? The table below organizes income levels (per year) and gender.

	Under $10,000 (L)	Between $10,000 and $40,000 (ML)	Between $40,000 and $100,000 (MH)	Over $100,000 (H)	Totals
Male (M)	15	64	37	61	177
Female (F)	31	73	14	58	176
Totals	46	137	51	119	353

   a. Are the events being male and making over $100,000 independent? Show your work.

$P(M) \cdot P(+100{,}000) = P(m \text{ and } +100{,}000)$

$\frac{177}{353} \cdot \frac{119}{353} = \frac{61}{354}$

$.169 \neq .173$ not independent

   b. Are the events being a male and making less than $10,000 independent? Justify your answer.

$P(m) \cdot P(<10{,}000) = P(m \text{ and } <10{,}000)$

$\frac{177}{353} \cdot \frac{46}{353} = \frac{15}{353}$

$.065 \neq .042$ not independent

4. DataTech Systems has 415 employees working in non-management positions. A total of 32 non-management employees have been working at the company for more than 5 years. The company is implementing a new management training program. Out of their 415 non-management employees, 105 applied to the new program. Of those 105 employees, 21 had been working for the company for more than 5 years.

   Let *F* represent the event that a randomly selected non-management employee has been with the company more than 5 years. Let *M* represent the event that a non-management employee has applied to the new management training program.

   Are the events F and M independent? Explain your reasoning.