NOTES: Proving Parallelograms and Rectangles on a Coordinate Plane

PARALLELOGRAMS ON THE COORDINATE PLANE

Objectives:

- Show that a quadrilateral is a parallelogram on the coordinate plane
- Identify and verify parallelograms

DISTANCE FORMULA:	MIDPOINT FORMULA:	SLOPE FORMULA:
$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$	$(x_m, y_m) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$	$m = \frac{y_2 - y_1}{x_2 - x_1}$

FORMULAS & THE COORDINATE PLANE	
FORMULA	WHEN TO USE IT
Distance Formula: $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$	To determine whether... • Sides are congruent length • Diagonals are congruent
Midpoint Formula: $(x_m, y_m) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$	To determine... • The coordinates of a midpoint of a side • Whether diagonals bisect each other
Slope Formula: $m = \frac{y_2 - y_1}{x_2 - x_1}$	To determine whether... • Opposite sides are parallel • Diagonals are perpendicular $m_1 \cdot m_2 = -1$ • Sides are perpendicular right angles

QUADRILATERAL	PROVE:
PARALLELOGRAM	• Both pairs of opposite sides are parallel • Both pairs of opposite sides are congruent • One pair of opposite sides are parallel and congruent • Diagonals bisect each other
RECTANGLE	First prove it's a parallelogram, and then prove... • The diagonals are congruent • Two consecutive sides of the parallelogram are perpendicular

Proving a Quadrilateral is a Parallelogram

Method: Show both pairs of opposite sides are equal by calculating the distances of all four sides.

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1) Plot and label each point. A(2, 4), B(7, 9), C(6, 3), and D(1, -2)

Prove it!

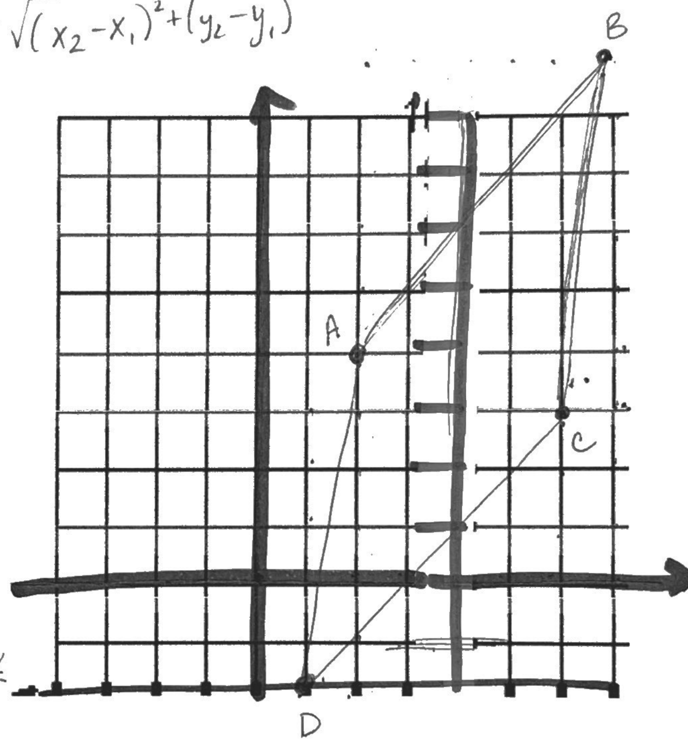
Find the **length** of each side to the nearest tenth. $d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

AB = $\sqrt{50}$ $d=\sqrt{(7-2)^2+(9-4)^2}$ $\sqrt{25+25}=\sqrt{50}$

BC = $\sqrt{37}$ $d=\sqrt{(6-7)^2+(3-9)^2}$ $\sqrt{1+36}=\sqrt{37}$

DC = $\sqrt{50}$ $d=\sqrt{(1-6)^2+(-2-3)^2}$ $\sqrt{25+25}=\sqrt{50}$

DA = $\sqrt{37}$ $d=\sqrt{(1-2)^2+(-2-4)^2}=$ $\sqrt{1+36}=\sqrt{37}$



- What conclusions can you make? (Hint: are any sides the same length)

$\overline{AB}\cong\overline{DC}$, $\overline{BC}\cong\overline{DA}$ opposite sides $\cong$

Find the **slope** of each side.

$m=\frac{\text{rise}}{\text{run}}$

Slope of AB = $\frac{5}{5}=1$

Slope of DC = $\cancel{\frac{-6}{-1}=6}$ $\frac{5}{5}=1$

same

Slope of BC = $\frac{-6}{-1}=6$

Slope of AD = $\frac{-6}{-1}=6$

same

Parallel sides
$\overline{AB}\parallel\overline{DC}$
$\overline{BC}\parallel\overline{AD}$

- What conclusions can you make? (Hint: are any sides parallel? Perpendicular ?)

Based on my answers above, I have proven this shape to be a Parallelogram
because...

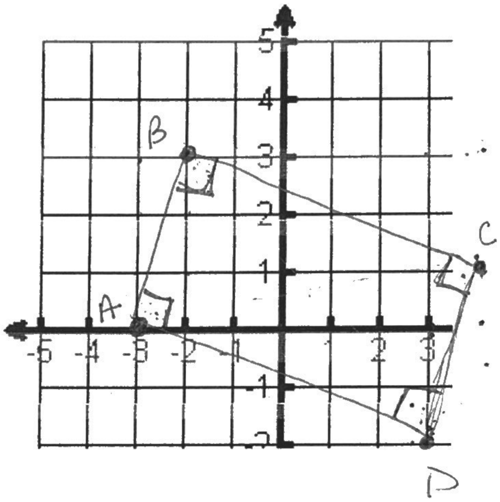
Proving a Quadrilateral is a Rectangle

Method: First, prove the quadrilateral is a parallelogram, then that the diagonals are congruent.

2) Plot and label each point. A(-3, 0), B(-2, 3), C(4, 1), and D(3, -2)

Prove it!

Find the **length** of each side to the nearest tenth.



AB = $\sqrt{10}$ $d = \sqrt{(-2+3)^2 + (3-0)^2}$ $\sqrt{1+9} = \sqrt{10}$

BC = $\sqrt{40}$ $d = \sqrt{(4+2)^2+(1-3)^2}$ $\sqrt{36+4} = \sqrt{40}$

DC = $\sqrt{10}$ $d = \sqrt{(3-4)^2+(-2-1)^2} =$ $\sqrt{1+9} = \sqrt{10}$

DA = $\sqrt{40}$ $d = \sqrt{(3+3)^2+(-2-0)^2}$ $\sqrt{36+4} = \sqrt{40}$

- What conclusions can you make? (Hint: are any sides the same length)

opposite sides Congruent $\overline{AB} \cong \overline{DC}$, $\overline{BC} \cong \overline{DA}$

Calculate the Distance of the Diagonals.

AC = $\sqrt{50}$ $d = \sqrt{(4+3)^2+(1-0)^2}$ $\sqrt{49+1} = \sqrt{50}$

BD = $\sqrt{50}$ $d = \sqrt{(3+2)^2+(-2-3)^2}$ $\sqrt{25+25} = \sqrt{50}$

Perpendiclar lines of 2 consecutive ∡'s. ✓

$m\overline{AD} = \frac{2}{-6} = -\frac{1}{3}$

$m\overline{AB} = \frac{3}{1} = 3$ $\}$ $-\frac{1}{3} \cdot 3 = -1$

$m\overline{BC} = \frac{-2}{6} = -\frac{1}{3}$ $\}$ ✓

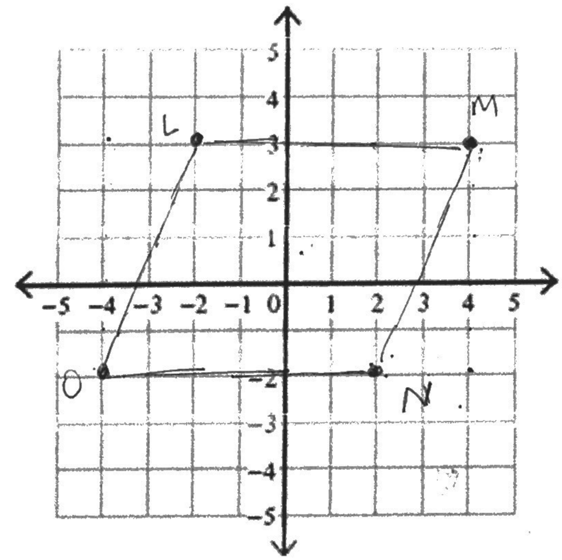
- What conclusions can you make? (Hint: are any sides parallel? Perpendicular ?)

$\overline{AC} \cong \overline{BD}$ Rectangle has $\cong$ diagonals $\}$ all angles are Right ∡'s $\overline{AD} \perp \overline{AB}$, $\overline{AB} \perp \overline{BC}$

Based on my answers above, I have proven this shape to be a Rectangle

because...

Prove that the quadrilateral with the coordinates L(-2,3), M(4,3), N(2,-2) and O(-4,-2) is a parallelogram.

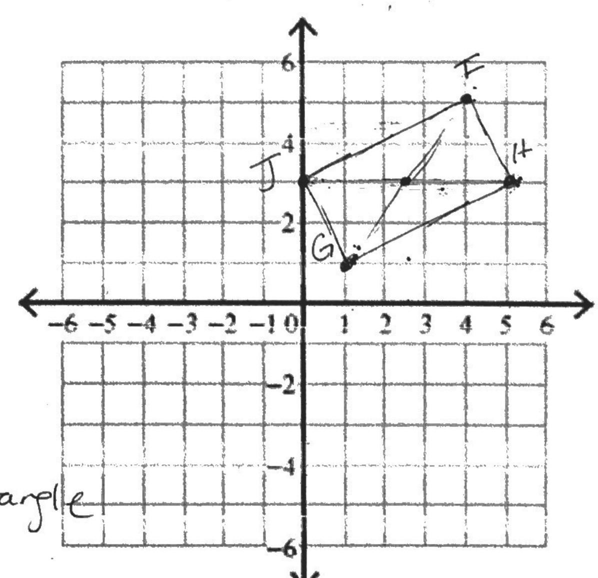


1. $\overline{LM} \parallel \overline{ON}$ $m = \frac{0}{6} = 0$

2. $\overline{LO} \parallel \overline{MN}$ $m = \frac{5}{2}$ $m = \frac{+5}{+2}$

Parallelogram because opposite sides are parallel! ☺

Prove a quadrilateral with vertices G(1,1), H(5,3), I(4,5) and J(0,3) is a rectangle.



$JH = \sqrt{(0-5)^2 + (3-3)^2}$
$\sqrt{25+0} = \sqrt{25} = 5$

$GI = \sqrt{(4-1)^2 + (5-1)^2} =$
$\sqrt{9+16} = \sqrt{25} = 5$

2 Diagonals are $\cong$, Rectangle

Midpoint JH $\left(\frac{5+0}{2}, \frac{3+3}{2}\right) = \left(\frac{5}{2}, \frac{6}{2}\right) = (2.5, 3)$

Midpoint GI $\left(\frac{1+4}{2}, \frac{1+5}{2}\right) = \left(\frac{5}{2}, \frac{6}{2}\right) = (2.5, 3)$

1 Diagonals Bisect each other, Parallelogram