Learning Task: Conditional Probability

The retail and service industries are one aspect of modern society where probability's relevance can be seen. By studying data on their own service and their clientele, businesses can make informed decisions about how best to move forward in changing economies. Below is a table of data collected over a weekend at a local ice cream shop, Frankie's Frozen Favorites. The table compares a customer's flavor choice to their cone choice.

Frankie's Frozen Favorites	Chocolate	Butter Pecan	Fudge Ripple	Cotton Candy	
Sugar Cone	36	19	34	51	140
Waffle Cone	35	56	35	24	150
	71	75	69	75	290

1. By looking at the table, but without making any calculations, would you say that there is a relationship between flavor and cone choice? Explain.

2. Find the following probabilities. Round all answers to the hundredths place.

a. $P(Sugar\ Cone)$ **b.** $P(Waffle\ Cone)$ **c.** $P(Chocolate)$

$\frac{140}{290} = 14/29 = \boxed{.48}$

d. $P(BP)$ **e.** $P(FR)$ **f.** $P(CC)$

$75/290 = 15/58 = \boxed{.26}$

3. By considering ONLY the people that prefer Fudge Ripple ice cream, what is the probability that they will choose a waffle cone?

↓ 69 $\frac{35}{69}$

In the above question, you have calculated a "**conditional probability**" because you are considering a specific group. **The "conditional" part is the denominator of the fraction, rather than the table total. Using notation, question 3 can be rewritten as** $P(waffle\ cone \mid FR)$.

Deter. denom.

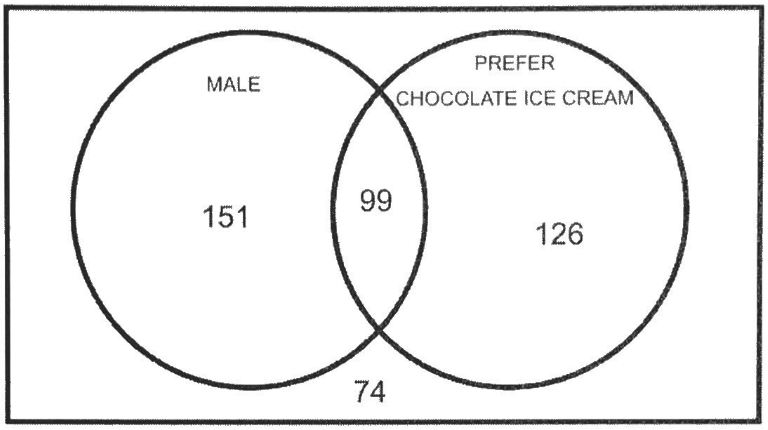
4. Find the probabilities for the following questions:

a. $P(sugar\ cone \mid chocolate) =$ $36/71 = \boxed{.51}$

b. $P(BP \mid sugar\ cone) =$ $\frac{19}{140} = \boxed{.14}$

c. $P(CC \mid waffle\ cone) =$ $\frac{24}{150} = \boxed{.16}$

A survey was conducted of both male and females as to whether they prefer chocolate or vanilla ice cream. The results of the survey are summarized in the following Venn diagram. Use this Venn Diagram to complete problems 3-5.



3. Use the Venn diagram to complete the below two way frequency table.

	Males	**Females**	**Total**
Vanilla	151	74	225
Chocolate	99	126	225
Total	250	200	450

4. Let Event A be the person surveyed is male and let Event B be the person surveyed prefers chocolate ice cream. Find the following:

a. P(A) male

$\frac{250}{450} = \boxed{.56}$

b. P(B) chocolate

$225/450 = \boxed{.50}$

c. P(A∪B) $\boxed{.84}$

$\frac{250 + 225 - 99}{450} = \frac{376}{450}$

d. P(A∩B)

$\frac{99}{450} = \boxed{.22}$

e. P(A') not male

$\frac{126+74}{450} = \frac{200}{450} = \boxed{.44}$

f. $P(\overline{A \cup B})$

not or = .84 $\quad 1 - .84 = \boxed{.16}$

g. P(A|B)

$\frac{P(A \text{ and } B)}{P(B)} = \frac{99}{225} = \boxed{.44}$

h. P(B|A)

$\frac{P(B \text{ and } A)}{P(A)} = \frac{99}{250} = \boxed{.40}$

5. Which probability in #4 corresponds to the probability that a person surveyed is female?

P(A') = not male = female (e)

Guided Practice:

1. All of the upperclassmen (juniors and seniors) at a high school were classified according to grade level and response to the question "How do you usually get to school?" The resulting data are summarized in the two-way table below. **Rewrite all questions using the proper notation.** Round all answers to the hundredths place.

	Car	Bus	Walk	Totals
Juniors	96	122	56	274
Seniors	184	58	30	272
Totals	280	180	86	546

a. If an upperclassman at this school is selected at random, what is the probability that he or she is a junior?

$274/546 = \boxed{.50}$

b. If an upperclassman at this school is selected at random, what is the probability that this student usually takes a bus to school?

$180/546 = \boxed{.33}$

c. If a randomly selected upperclassman says he or she is a junior, what is the probability that he or she usually walks to school?

$\frac{56}{274} = \boxed{.20}$

2. Fill in the following two-way table that represents the GPAs of the football and basketball players and calculate the probabilities. Round all answers to the hundredths place.

	A (A)	B/C (BC)	D/F (DF)	Total
Football (F)	4	51	10	65
Basketball (B)	2	12	6	20
Total	6	63	16	85

a. P(F|A) = $\frac{4}{6} = .67$

b. P(BC|B) = $\frac{12}{20} = .60$

c. P(B|DF) = $\frac{6}{16} = .38$

Learning Task: Conditional Probability using a Formula

$$P(A|B)=\frac{P(A \text{ and } B)}{P(B)}$$

Say-No-To-Smoking campaigns are vigilant in educating the public about the adverse health effects of smoking cigarettes. This motivation to educate the public has its beginnings in data analysis. Below is a table that represents a sampling of 500 people. Distinctions are made on whether or not a person is a smoker and whether or not they have ever developed lung cancer. Each number in the table represents the number of people that satisfy the conditions named in its row and column.

	Has been a smoker for 10+	**Has not been a smoker**	
Has not developed lung cancer	202	270	472
Has developed lung cancer	23	5	28
	225	275	500

a. Using the table, show that $P(Smoker|Lung\ Cancer) = \frac{23}{28}$

b. Using the conditional probability formula above, show that $P(Smoker|Lung\ Cancer) = \frac{23}{28}$

$$P(S|LC) = \frac{P(\text{Smoker And Lung Cancer})}{P(\text{Lung Cancer})}$$

c. Using the same approach, show that the conditional probability formula works for $P(\bar{S}|\bar{L})$.

$$P(\bar{S}|\bar{L}) = \frac{P(\text{not smoker and not lung cancer})}{P(\text{not lung Cancer})} = \frac{270}{472}$$

$$P(A|B) = \frac{P(A \text{ and } B)}{P(B)}$$

Guided Practice: Round all answers to the hundredths place.

1. For two events S and Q it is known that $P(Q) = .45$ and $P(S \cap Q) = .32$. Find $P(S|Q)$.

$$P(S|Q) = \frac{P(S \text{ and } Q)}{P(Q)} = \frac{.32}{.45} = \boxed{.71}$$

2. For the events X and Y it is known that $P(Y) = \frac{1}{5}$ and $P(X \cap Y) = \frac{2}{15}$. Find $P(X|Y)$.

$$P(x|y) = \frac{P(x \text{ and } y)}{P(y)} = \frac{2/15}{1/5} = .67 = \boxed{2/3}$$

3. For two events B and C it is known that $P(C|B) = .61$ and $P(C \cap B) = .48$. Find $P(B)$.

$$P(C|B) = \frac{P(C \text{ and } B)}{P(B)}$$

$$\frac{.61}{1} = \frac{.48}{B} \qquad \frac{\cancel{.61}B}{\cancel{.61}} = \frac{.48}{.61} = \boxed{.79}$$

4. For the events V and W it is known that $P(W) = \frac{2}{9}$ and $P(V|W) = \frac{2}{11}$. Find $P(V \cap W)$.

$$P(V|W) = \frac{P(V \text{ and } W)}{P(W)}$$

$$\frac{\frac{2}{11}}{1} = \frac{X}{2/9} \qquad X = \left(\frac{2}{11}\right)\left(\frac{2}{9}\right) = \boxed{\frac{4}{99}}$$

5. For two events G and H it is known that $P(H|G) = \frac{5}{14}$ and $P(H \cap G) = \frac{1}{3}$. Explain why you cannot determine the $P(H)$.

$$P(H|G) = \frac{P(H \text{ and } G)}{P(G)}$$

$$\frac{5}{14} = \frac{\frac{1}{3}}{P(G)}$$

P(H) is not in the formula. Therefore you can't determine it's value.