Converting General Form to Standard Form

Now **complete the square** to rewrite the following equations in standard form. An example is shown below:

$Ax^2 + By^2 + Cx + Dy + F = 0$

Example:

$$x^2 + y^2 - 8x + 6y - 56 = 0$$

Move the constant

$$+56 \quad +56$$

$$x^2 + y^2 - 8x + 6y = 56$$

Group like terms together

$$(x^2 - 8x) + (y^2 + 6y) = 56$$

Take half of the middle coefficient, square it and add it to both sides

$$\left(x^2 - 8x + \left(\frac{-8}{2}\right)^2\right) + \left(y^2 + 6y + \left(\frac{6}{2}\right)^2\right) = 56 + \left(\frac{-8}{2}\right)^2 + \left(\frac{6}{2}\right)^2$$

$\sqrt{x^2 - 8x + \sqrt{16}}$

$16x^2 | -8x$

$-4 . -4 | -4 + -4 = -8$

$(x-4)^2$

	x	-4
x	x^2	$-4x$
-4	$-4x$	16

$= x^2 - 8x + 16$

Factor as a binomial squared, and simplify the squared terms

$$(x-4)^2 + (y+3)^2 = 56 + (-4)^2 + (3)^2$$

$$(x-4)^2 + (y+3)^2 = 56 + 16 + 9$$

Simplify

$$(x-4)^2 + (y+3)^2 = 81$$

Radius = r

Center

$(x-h)^2 + (y-k)^2 = r^2$ $\quad (h, k)$

Guided Practice: Change the following equations to standard form. Then, identify the radius and center.

1. $x^2 + y^2 + 2x - 10y = -22$

$\sqrt{x^2 + 2x + \sqrt{1}} + \sqrt{y^2 - 10y + \sqrt{25}} = -22 + 1 + 25$

$\frac{2}{2} = (1)^2 \qquad \frac{-10}{2} = (-5)^2$

$(x+1)^2 + (y-5)^2 = 4$

Standard Form: $(x+1)^2 + (y-5)^2 = 4$

Radius: $\sqrt{4}$ 2 Center: $(-1, 5)$

2. $x^2 + y^2 + 14y + 24 = 0$

$-24 \quad -24$

$x^2 + \sqrt{y^2 + 14y + \sqrt{49}} = -24 + 49$

$\frac{14}{2} = (7)^2$

x^2

$x^2 + (y+7)^2 = 25$

Standard Form: $x^2 + (y+7)^2 = 25$

Radius: $\sqrt{25}$ 5 Center: $(0, -7)$

On your own: Rewrite in Standard form for a circle: $(x-h)^2+(y-k)^2=r^2$. Then, state the radius and center.

1. $x^2+y^2+8x=84$

$x^2+8x+16+y^2=84+16$

$\frac{8}{2}=(4)^2$

$(x+4)^2+y^2=100$

Center: $(-4,0)$

radius: $\sqrt{100}=10$

2. $x^2+y^2-18y+65=0$

$-65\ \ -65$

$x^2+y^2-18y+81=-65+81$

$\frac{-18}{2}=(-9)^2$

$x^2+(y-9)^2=16$

Center: $(0,9)$

Radius: 4

3. $x^2+y^2+20x-26y+268=0$

$-268\ \ -268$

$x^2+20x+100+y^2-26y+169=-268+100+169$

$\frac{20}{2}=(10)^2$ $\quad \frac{-26}{2}=(-13)^2$

$(x+10)^2+(y-13)^2=1$

Center: $(-10,13)$

Radius: $\sqrt{1}=1$

4. $x^2+y^2+14x-22y+134=0$

$-134\ \ -134$

$x^2+14x+49+y^2-22y+121=-134+49+121$

$\frac{14}{2}=(7)^2$ $\quad \frac{-22}{2}=(-11)^2$

$(x+7)^2+(y-11)^2=36$

Center: $(-7,11)$

Radius: $\sqrt{36}=6$