Area and Area of a Sector

Formulas:

Area of a Circle: πr^2

Area of a Sector: $\dfrac{\pi r^2 \theta}{360}$, where θ is the <u>Central Angle</u>

Example 1: Find the exact area of the circle.

2 in.
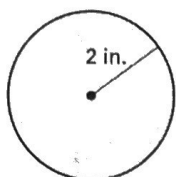
$\pi(2)^2 = \boxed{4\pi \text{ exact}}$

$4\pi \approx \boxed{12.57 \text{ estimate}}$

Example 2:

a. The area of a circle is 58 square inches. Find the <u>radius</u>.

$A = \pi r^2$

$\dfrac{58}{\pi} = \dfrac{\pi r^2}{\pi}$ $\sqrt{18.46} = \sqrt{r^2}$

$\boxed{r \approx 4.30}$

b. The area of a circle is 37 square meters. Find the <u>radius</u>.

$\dfrac{37}{\pi} = \dfrac{\pi r^2}{\pi}$ $116.24 \approx r^2$

$\boxed{r \approx 10.78}$

c. The area of a circle is 106 square centimeters. Find the <u>diameter</u>.

$\dfrac{106}{\pi} = \dfrac{\pi r^2}{\pi}$ $r^2 \approx 33.74$ $d = 2(5.81)$

$r \approx 5.81$ $\boxed{d \approx 11.62}$

d. The area of a circle is 249 square feet. Find the <u>diameter</u>.

$\dfrac{249}{\pi} = \dfrac{\pi r^2}{\pi}$ $r^2 \approx 79.26$ $d = 2(8.90)$
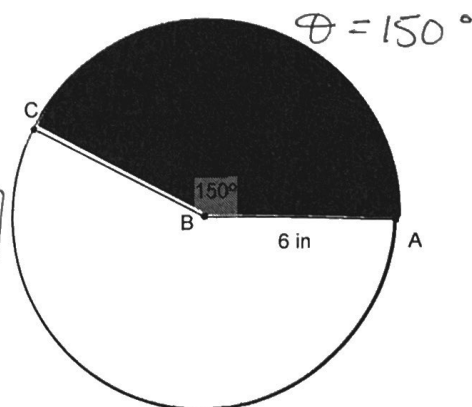
$r \approx 8.90$ $\boxed{d \approx 17.81}$

Example 3: Given the diagram to the right to find the area of the shaded sector. $m\angle CBA = 150°$
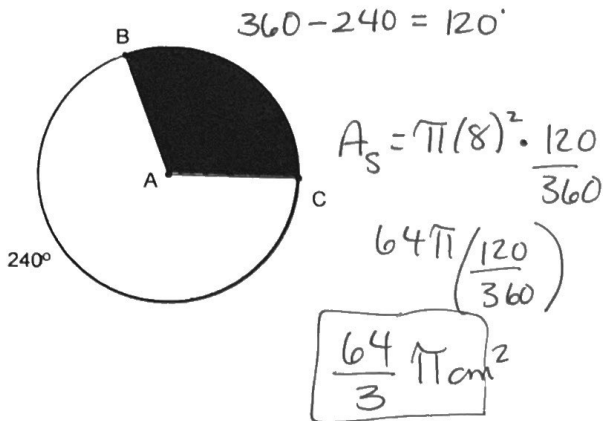
$A = \pi(6)^2$

$A = 36\pi$

$\theta = 150°$

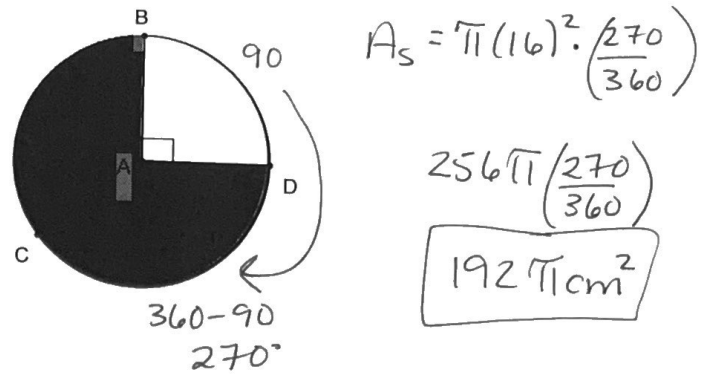Area Sector $= 36\pi \cdot \left(\dfrac{150}{360}\right) = \boxed{15\pi \text{ in}^2}$

exact

150°

B   6 in   A

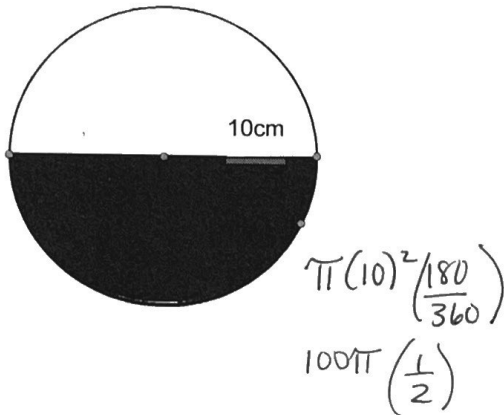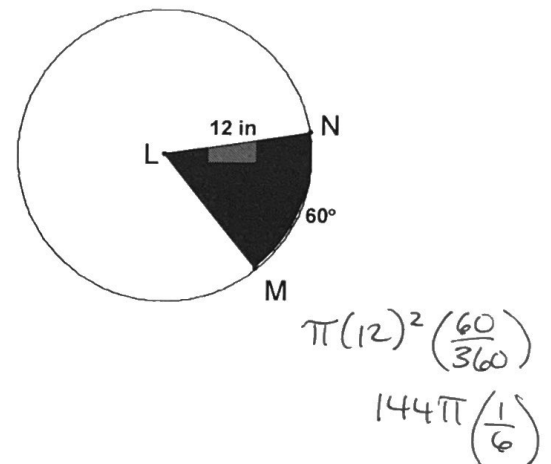**Example 4:** Find the area of the shaded region.

a. r = 8 cm

$360 - 240 = 120°$

$A_s = \pi(8)^2 \cdot \dfrac{120}{360}$

$64\pi\left(\dfrac{120}{360}\right)$

$\boxed{\dfrac{64}{3}\pi \, cm^2}$

240°

b. r = 16 cm

90

$A_s = \pi(16)^2 \cdot \left(\dfrac{270}{360}\right)$

$256\pi\left(\dfrac{270}{360}\right)$

$\boxed{192\,\pi\,cm^2}$

$360 - 90$
$270°$

Example 5: Find the area of the **shaded** region.

a.

10cm

$\pi(10)^2\left(\dfrac{180}{360}\right)$

$100\pi\left(\dfrac{1}{2}\right)$

Shaded area = $\underline{50\pi \, cm^2}$

b.

12 in   N

L

60°

M

$\pi(12)^2\left(\dfrac{60}{360}\right)$

$144\pi\left(\dfrac{1}{6}\right)$

Shaded area = $\underline{24\pi \, in^2}$

c.

300°

4ft

$360 - 300$
$60°$

$\pi(4)^2 \cdot \left(\dfrac{60}{360}\right)$

$16\pi\left(\dfrac{1}{6}\right)$
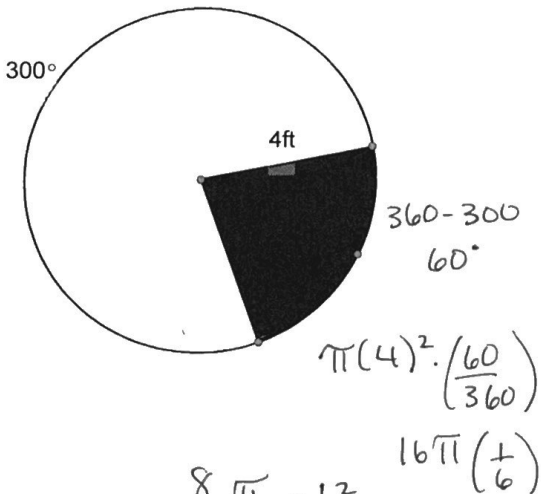
Shaded area = $\underline{\dfrac{8}{3}\pi \, ft^2}$

d. radius = 10 cm.

10 cm

80°

A

80°   B

$360 - 80° = 280°$

$\pi(10)^2\left(\dfrac{280}{360}\right)$
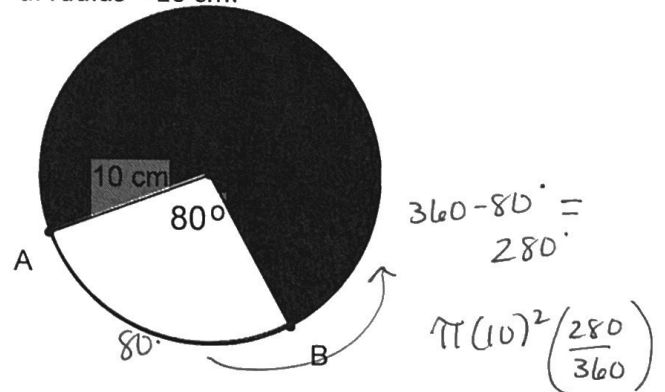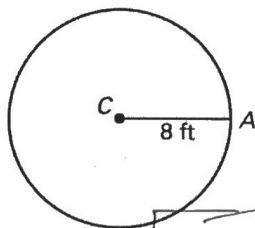
$\dfrac{700\pi}{9}$

Shaded area = $\underline{\dfrac{700\pi}{9}} \, cm^2$

Skills Practice: Area and Area of a Sector

1. Find the exact area of the circle.

a.

C 8 ft A

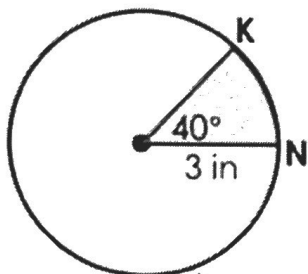$$A = \pi(8)^2 = \boxed{64\pi \, ft^2}$$

b.

A $\frac{1}{2}$ yd C
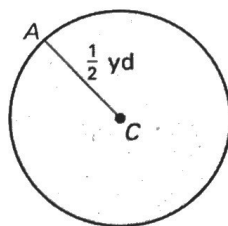
$$A = \pi\left(\frac{1}{2}\right)^2 = \boxed{\frac{1}{4}\pi \, yd^2}$$
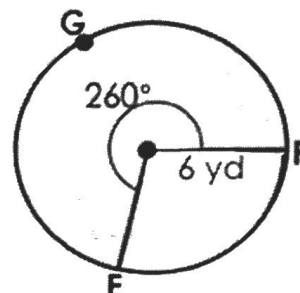
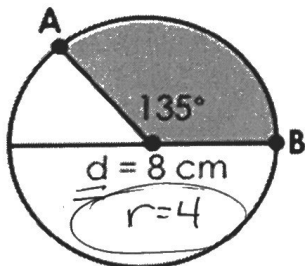2. Find the area of each sector below. Leave all answers in terms of pi.

a.

K 40° 3 in N

$$\pi(3)^2\left(\frac{40}{360}\right) = \boxed{\pi \, in^2}$$

b.

G 260° 6 yd F E

$$\pi(6)^2\left(\frac{260}{360}\right) = \boxed{26\pi \, yd^2}$$

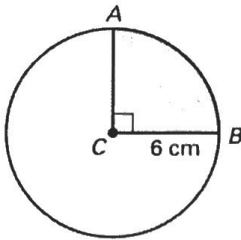c.

A 135° d = 8 cm $r=4$ B

$$\pi(4)^2\left(\frac{135}{360}\right) = \boxed{6\pi \, cm^2}$$

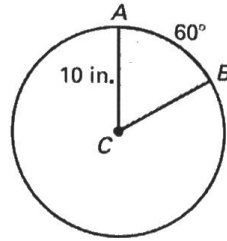**Find the areas of the sectors formed by angle *ACB*.**

3.

A

C 6 cm B

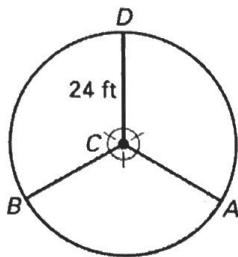$\pi(6)^2\left(\frac{90}{360}\right)$

$\boxed{9\pi \,cm^2}$

5.

A 60°

10 in. B

C

$\pi(10)^2\left(\frac{60}{360}\right)$

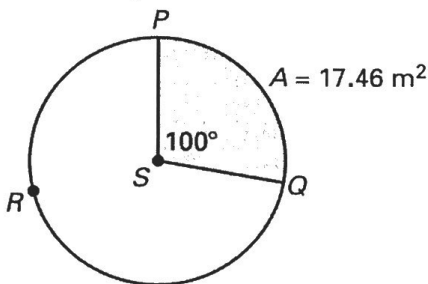$\boxed{\frac{50\pi}{3}\,in^2}$

4.

D

24 ft

C

B A

$360/_3 = 120$

$\pi(24)^2\left(\frac{120}{360}\right) = \boxed{192\pi \,ft^2}$

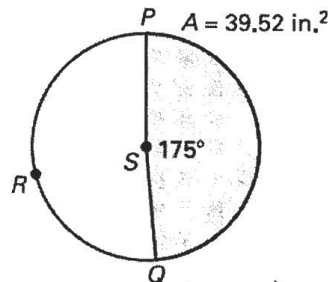**Use the diagram to find the indicated measure.**

6. Find the area of ⊙S

P

A = 17.46 m²

100°

S

R Q

$A_s = \pi r^2\left(\frac{\theta}{360}\right)$

$\dfrac{17.46 = \pi r^2\left(\frac{100}{360}\right)}{5/_{18}\pi} \quad \dfrac{}{5/_{18}\pi}$

$r^2 \approx 20.01 \quad r \approx 4.47$

$A_\odot = \pi(4.47)^2 = $

$\boxed{19.98\pi \,m^2}$

7. Find the area ⊙S

P A = 39.52 in.²

S 175°

R

Q

$39.52 = \pi r^2\left(\frac{175}{360}\right)$

$25.88 = r^2$

$5.09 \approx r$

$A_\odot = \pi(5.09)^2$

$= \boxed{25.91\pi \,in^2}$

8. Find the area ⊙S

A = 20.89 ft²

P

78° S

Q

R

$360 - 78 = $

$282°$
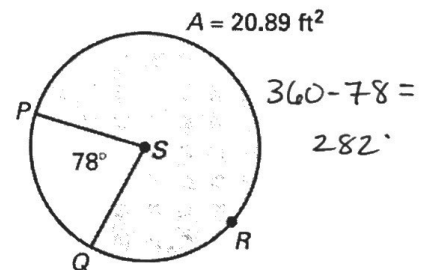
$20.89 = \pi r^2\left(\frac{282}{360}\right)$

$8.49 = r^2$

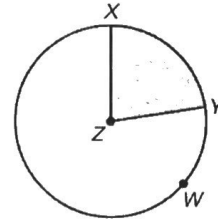$r \approx 2.91$

$A_\odot = \pi(2.91)^2$

$\boxed{8.49\pi \,ft^2}$

9. The area of ⊙Z is 124.44 square centimeters. The area of sector *XZY* is 28 square centimeters. Find the indicated measure.

a. Radius of ⊙Z
$$A = \pi r^2$$
$$124.44 = \pi r^2$$
$$39.61 = r^2 \qquad r \approx 6.29 \, cm$$

b. Circumference of ⊙Z
$$2\pi(6.29) = 39.52 \, cm$$

c. $m\widehat{XY}$
$$A_s = \pi r^2 \frac{\theta}{360°}$$
$$28 = \pi(6.29)^2 \cdot \frac{\theta}{360}$$
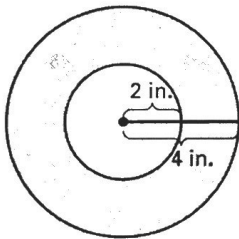$$\theta \approx 81.1°$$

d. Length of $\widehat{XY}$ (omit)

e. Perimeter of shaded region (omit)

f. Perimeter of unshaded region (omit)

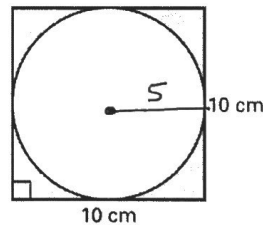**10.** Find the area of the shaded region.

a.

2 in.

4 in.

Big - Small

$16\pi - 4\pi$

$\boxed{12\pi \, in^2}$
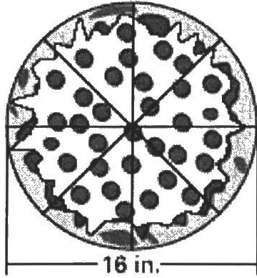
b.

5

10 cm

10 cm

Big - Small
Square - circle
$$(10)^2 \qquad \pi(5)^2$$
$$\boxed{100 - 25\pi \, cm^2}$$
$$\boxed{or \; 21.46 \, cm^2}$$

11. **Pizza** A pizza is cut into 8 congruent pieces as shown. The diameter of the pizza is 16 inches. Find the area of one piece of pizza.
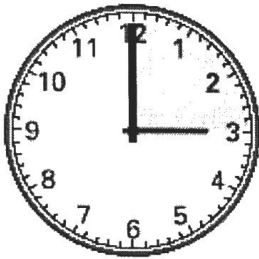
$r = 8 in$

16 in.

$$\frac{360}{8} = 45°$$

$$A_s = \pi(8)^2 \left(\frac{45}{360}\right) = \boxed{8\pi \ in^2}$$

12. **Clock** A wall clock has an area of 452.39 inches. Find the diameter of the clock. Then find the area of the sector formed when the time is 3:00 as shown.

$$A = \pi r^2$$
$$452.39 = \pi r^2$$
$$144.00 = r^2$$
$$12 = r$$

$$A_s = \pi(12)^2 \left(\frac{90}{360}\right) = \boxed{36\pi \ in^2}$$

$\frac{360}{60} = 6$

$6 \times 15 = 90°$