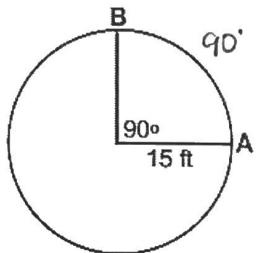# Calculating Arc Length

Arc Length is a fraction of the circle's circumference and is measured in linear units.

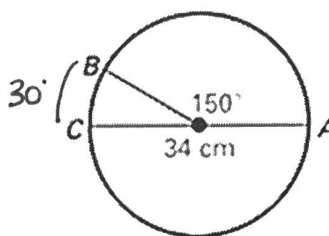| Arc Length |
|---|
| $2\pi r \cdot \dfrac{\theta}{360} = \dfrac{2\pi r\theta}{360}$, where θ is the central angle (or intercepted arc measure |

Example: Find the length of arc BA.

$AL = \dfrac{2\pi(15)\ 90}{360} = 7.5$

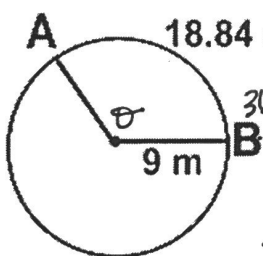$AL = \dfrac{15}{2}\pi$ ft.

Example: Find the length of arc BC.

$180 - 150 = 30° = \theta$

$AL = \pi(34)\left(\dfrac{30}{360}\right)$

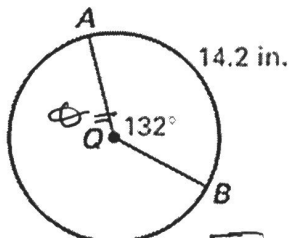$AL = \dfrac{17}{6}\pi$ cm

Example: Find the measure of arc AB.

$AL = \dfrac{2\pi r\theta}{360}$

$360 \cdot 18.84 = \dfrac{2\pi(9)\theta}{360} \cdot \dfrac{1}{360}$

$\dfrac{6782.4}{18} = \dfrac{18\pi\theta}{18}$

$\dfrac{376.8}{\pi} = \dfrac{\pi\theta}{\pi}$

$\boxed{119.9°} \approx$

Example: Find the radius of Circle Q.

$AL = \dfrac{2\pi r\theta}{360}$

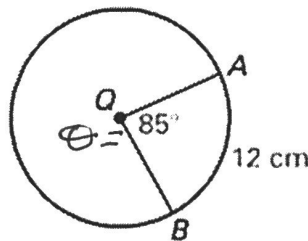$360 \cdot 142 = \dfrac{2\pi r(132)}{360} \cdot 360$

$\dfrac{5112}{132} = \dfrac{2\pi r(132)}{132}$

$\dfrac{38.73}{2} = \dfrac{2\pi r}{2}$

$19.36 = \dfrac{\pi r}{\pi}$

$\boxed{6.2\text{ in} = r}$

Example: Find the circumference of Circle Q.

$C = \pi d = 2\pi r$

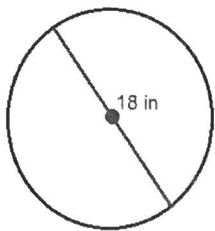$AL = \boxed{\dfrac{2\pi r\theta}{360}}$

$360 \cdot 12 = \dfrac{\boxed{2\pi r}(85)}{360} \cdot 360$

$\dfrac{4320}{85} = \dfrac{\boxed{2\pi r}(85)}{85}$

$50.8 = 2\pi r$

$\boxed{50.8\text{ cm} = C}$

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Use the formulas to answer the questions below. Be sure to leave all answers in terms of pi.
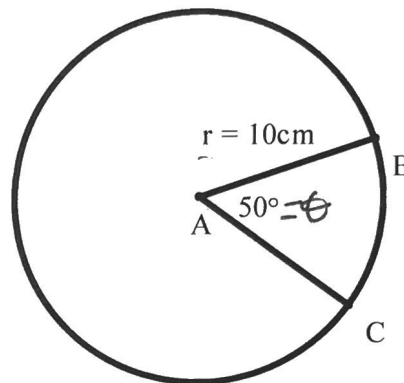
EXAMPLE 1: Find the circumference of the circle.

$C = d\pi$ or $2\pi r$

$C = 18\pi$ in

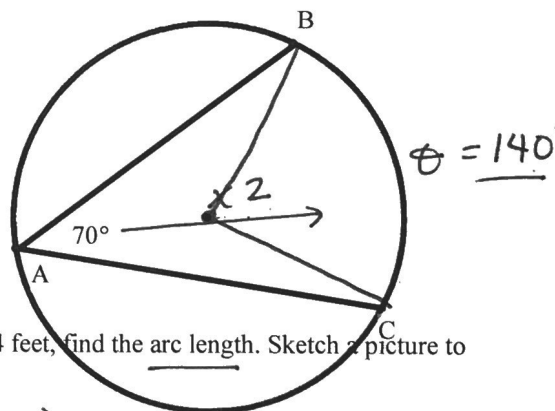**Example 2:** Use the diagram of the circle to find the arc length of BC.

$AL = \dfrac{2\pi r \, \theta}{360}$

$= \dfrac{2\pi(10)(50)}{360} = \boxed{\dfrac{25}{9}\pi \, cm}$

r = 10cm

50° = θ

Example 3: Use the diagram of the circle to find the arc length of BC with a radius of 4 inches.

$AL = \dfrac{2\pi(4)(140)}{360} = \boxed{\dfrac{28}{9}\pi \, in}$

θ = 140°

70°

Example 4: If a central angle measures 80° and the diameter of the circle measures 24 feet, find the arc length. Sketch a picture to help you solve the problem.
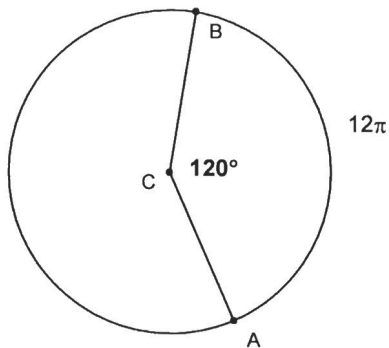
$\theta = 80°$
$d = 24 ft.$
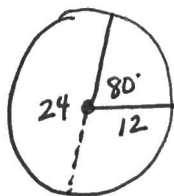
$AL = 2\pi(12)\left(\dfrac{80}{360}\right) = \boxed{\dfrac{16}{3}\pi \, ft.}$

Example 5: Use the formula that you have developed for arc length and find the **circumference** of the circle.

12π

120°

$AL = \dfrac{\boxed{2\pi r}\,\theta}{360}$

$_{360}\,12\pi = \dfrac{\boxed{2\pi r}(120)}{360}\,_{360}$

$\dfrac{4320\pi}{120} = \dfrac{\boxed{2\pi r}(120)}{120}$

$36\pi = 2\pi r$

$\boxed{36\pi = C}$

Radius

$\dfrac{36\pi}{2} = \dfrac{2\pi r}{2}$

$\dfrac{18\pi}{\pi} = \dfrac{\pi r}{\pi}$

$\boxed{18 = r}$

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