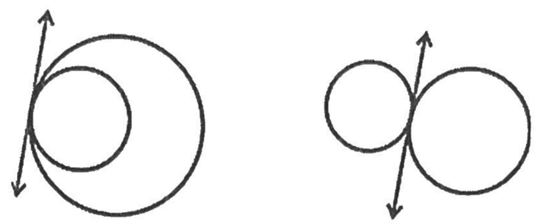
# Tangent and Chord Properties

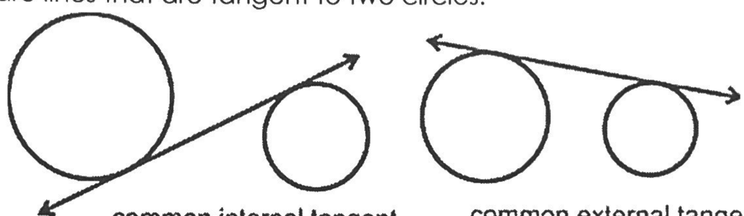
On Day 1, you learned that tangent lines intersect a circle in exactly one place. This leads to several theorems about tangent lines.

**Tangent Circles** are two coplanar circles that intersect at exactly one point. They may intersect internally or externally.

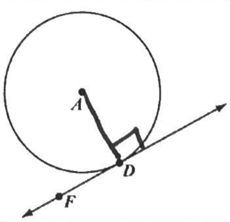
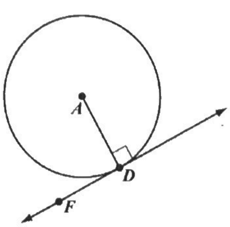


internally tangent  externally tangent

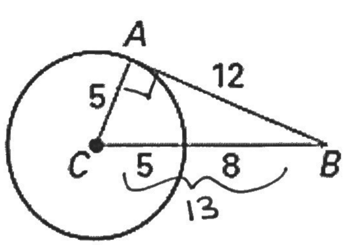
**Common Tangent Lines** are lines that are tangent to two circles.



common internal tangent  common external tangent

| Name | Theorem | Hypothesis | Conclusion |
|---|---|---|---|
| **Perpendicular Tangent Theorem** | If a line is tangent to a circle, then it is **perpendicular** to the radius drawn to the point of tangency. |  | Radius $\overline{AD}$ $\perp$ to the tangent $\overleftrightarrow{FD}$ |
| **Converse of Perpendicular Tangent Theorem** | If a line is **perpendicular** to a radius of a circle at a point on the circle, then the line is tangent to the circle. |  | |

**Example:** Is AB tangent to Circle C?




$a^2+b^2=c^2$

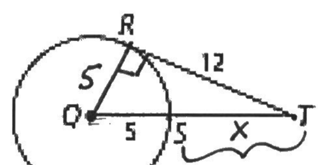
$(5)^2+(12)^2=(13)^2$

$25+144=169$

$169=169$

Yes!

**Example:** Find ST.




| | 5 | x |
|---|---|---|
| 5 | 25 | 5x |
| x | 5x | $x^2$ |

$25+10x+x^2$

$-144x^2+10x$

$(-8x\cdot 18x)$ | $-8x+18x$ $=10x$

$5^2+12^2=(5+x)^2$

$169=25+10x+x^2$

$-169 \quad -169$

$0=-144+10x+x^2$

$x^2+10x-144=0$

$(x-8)(x+10)=0$

$$a^2+b^2=c^2$$

$$(5)^2+(12)^2=(5+x)^2$$

$$\underbrace{25+144}$$

$$169=x^2+10x+25$$
$$-169 \qquad\qquad -169$$

$$0=x^2+10x-144$$

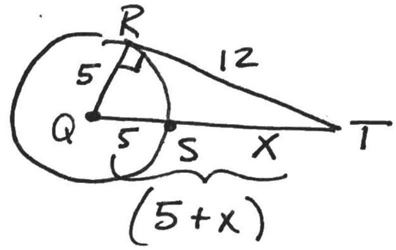
$$0=(x-8)(x+18)$$

$$x-8=0 \qquad x+18=0$$
$$+8 \quad +8 \qquad -18 \quad -18$$
$$x=8 \qquad x=\cancel{-18}$$

$$5+8=\boxed{13} \qquad 5+(-18)=-13$$

| | 5 | +x |
|---|---|---|
| 5 | 25 | 5x |
| +x | 5x | $x^2$ |

$$25+10x+x^2$$



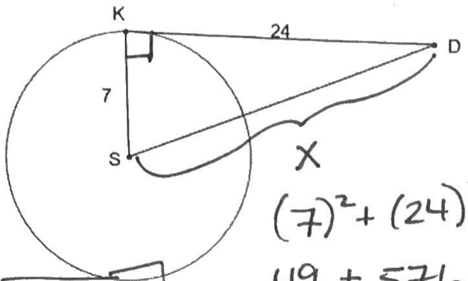

$-144$

$10x$

| $-144x^2$ | $+10x$ |
|---|---|
| $-1x \cdot 144x$ | $-1+144=143$ |
| $-2x \cdot 72x$ | $-2+72=70$ |
| $-3x \cdot 48x$ | |
| $-4x \cdot 36x$ | |
| $-6x \cdot 24x$ | |
| $-8x \cdot 18x$ | $-8+18=\underline{\underline{10}}$ |

Try:

1. If $\overline{KD}$ is tangent to circle S, find the length of $\overline{SD}$.



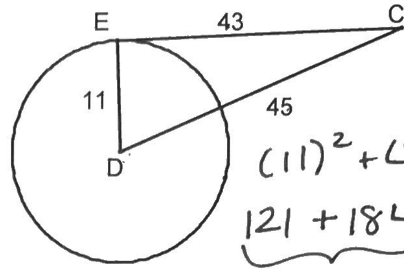

$(7)^2 + (24)^2 = (x)^2$

$49 + 576 = x^2$

$\sqrt{625} = \sqrt{x^2}$

$\boxed{x = 25}$

2. Determine if $\overline{EC}$ is tangent to circle D. Explain your answer.



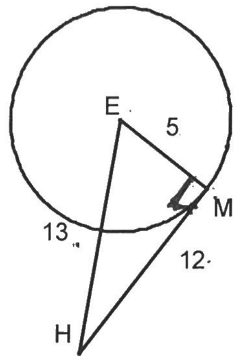

$(11)^2 + (43)^2 = (45)^2$

$\underbrace{121 + 1849}_{} = 2025$

$1970 \neq 2025$

No because $\overline{EC}$ does not form a right ∡ w/ the radius $\overline{ED}$.

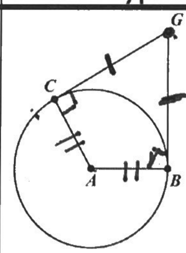
3. Is segment MH tangent to circle E? Justify your answer.



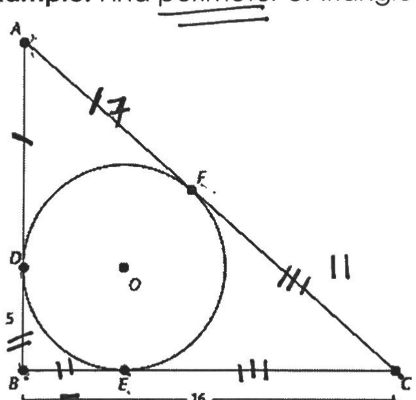

$5^2 + 12^2 = 13^2$

$\underbrace{25 + 144}_{} = 169$

$169 = 169$ ✓

| Name | Theorem | Hypothesis | Conclusion |
|---|---|---|---|
| **Tangent Segments Theorem** | If two segments are tangent to a circle from the same external point, then the segments are congruent. |  | $\overline{CB} \cong \overline{CG}$ |

**Example:** Find perimeter of triangle ABC.

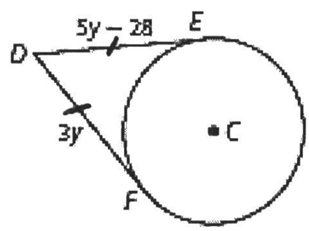



$16 - 5 = 11$

$P = 7 + 5 + 5 + 11 + 11 + 7$

$\boxed{46}$

**Example:** Find DF if you know that DF and DE are tangent to ⊙$C$.




$5y - 28 = 3y$

$-5y \qquad -5y$

$\frac{-28}{-2} = \frac{-2y}{-2}$

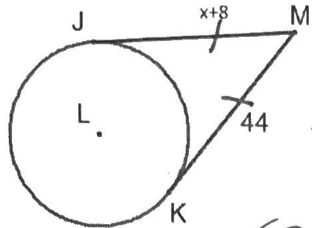
$y = 14$

$3(14) = \boxed{42}$

$5(14) - 28 = 42$

Try:

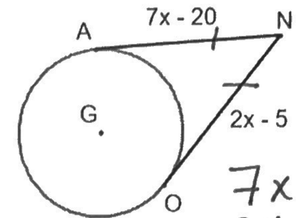
1. $\overline{JM}$ and $\overline{MK}$ are tangent to circle L.
   Find the value of x.




$x + 8 = 44$
$\cancel{-8} \quad -8$
$x = \boxed{36}$

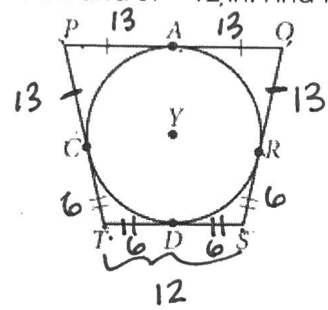
✓ $8 + (36) = 44$

2. $\overline{NA}$ and $\overline{NO}$ are tangent to circle G.
   Find the value of x.




$7x - 20 = 2x - 5$
$-2x \quad\quad -2x$
$5x - 20 = -5$
$+20 \quad +20$
$5x = 15$ $\boxed{x = 3}$

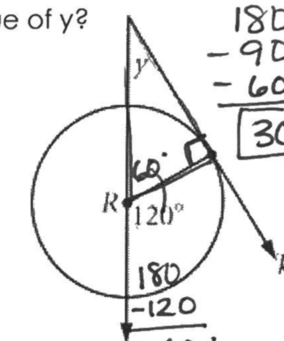
3. Quadrilateral POST is circumscribed about circle Y.
   OR = 13 in. and ST = 12 in. Find the perimeter of POST.




$4(13) + 4(6) = P$

$\boxed{P = 76 in}$

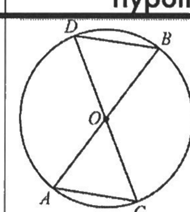
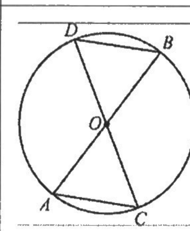
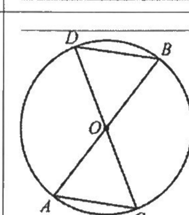
4. Ray k is tangent to circle R.
   What is the value of y?




180
−90
−60

$\boxed{30° = y}$

## Chord Properties

| Name | Theorem | Hypothesis | Conclusion |
|---|---|---|---|
| **Congruent Angle-Congruent Chord Theorem** | Congruent central angles have congruent chords. |  | |
| **Congruent Chord-Congruent Arc Theorem** | Congruent chords have congruent arcs. |  | |
| **Congruent Arc-Congruent Angle Theorem** | Congruent arcs have congruent central angles. |  | |