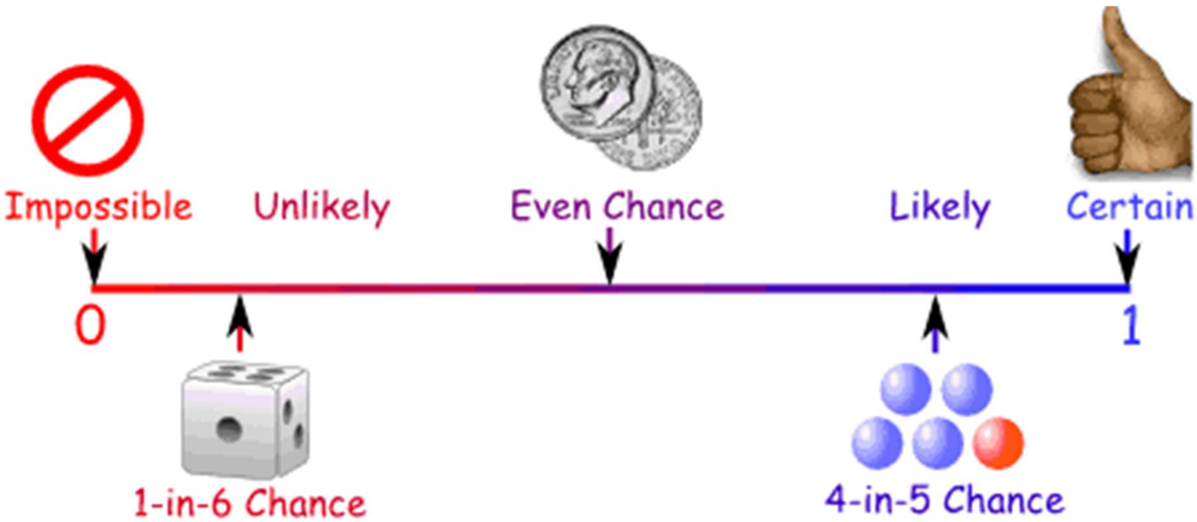
Unit 7: Probability



Standards Addressed:

Understand independence and conditional probability and use them to interpret data

MGSE9-12.S.CP.1 Describe categories of events as subsets of a sample space using unions, intersections, or complements of other events (*or, and, not*).

MGSE9-12.S.CP.2 Understand that if two events A and B are independent, the probability of A and B occurring together is the product of their probabilities, and that if the probability of two events A and B occurring together is the product of their probabilities, the two events are independent.

MGSE9-12.S.CP.3 Understand the conditional probability of A given B as P (A and B)/P(B). Interpret independence of A and B in terms of conditional probability; that is the conditional probability of A given B is the same as the probability of A and the conditional probability of B given A is the same as the probability of B.

MGSE9-12.S.CP.4 Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. *For example, use collected data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results*.

MGSE9-12.S.CP.5 Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. *For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer.*

Use the rules of probability to compute probabilities of compound events in a uniform probability model

MGSE9-12.S.CP.6 Find the conditional probability of A given B as the fraction of B's outcomes that also belong to A, and interpret the answer in context.

MGSE9-12.S.CP.7 Apply the Addition Rule, P(A or B) = P(A) + P(B) – P(A and B), and interpret the answers in context.

Warm Up: Introduction to Probability

For this task you will need a pair of six-sided dice. You are interested in the probability that one (or both) of the dice show odd values.

1. Roll your pair of dice 10 times, each time recording a success if one (or both) of the dice show an odd number and a failure if the dice do not show an odd number.

Number of Successes	Number of Failures

2. Based on your trials, what would you estimate the probability of two dice showing **at least** one odd number? Explain your reasoning.

3. You have just calculated an ***empirical probability***. 10 trials is generally sufficient to estimate the ***theoretical probability***, the probability that you expect to happen based upon fair chance. For instance, if you flip a coin ten times you expect the coin to land heads and tails five times apiece; in reality, we know this does not happen every time you flip a coin ten times.

a. A **lattice diagram** is useful in finding the theoretical probabilities for two dice thrown together. An incomplete lattice diagram is shown to the right. Each possible way the two dice can land, also known as an **outcome**, is represented as an ordered pair. (1, 1) represents each die landing on a 1, while (4, 5) would represent the first die landing on 4, the second on 5.

Complete the lattice diagram, which represents the **sample space** for the two dice thrown. The sample space for any probability model is a list of ALL possible outcomes.

Dice Lattice

(1,1)	(1,2)	(1,3)	(1,4)	(,)	(,)
(2,1)	(2,2)	(2,3)	(,)	(2,5)	(2,6)
(,)	(3,2)	(3,3)	(3,4)	(,)	(3,6)
(4,1)	(4,2)	(,)	(4,4)	(4,5)	(4,6)
(5,1)	(,)	(5,3)	(5,4)	(5,5)	(5,6)
(,)	(6,2)	(6,3)	(,)	(6,5)	(6,6)

b. **Explain** why the lattice diagram has to have 36 spaces to be filled.

c. Using your lattice, write the **set** of outcomes that the sum of two die rolled will be a 9.

d. Using your lattice, find the **probability** that the sum of two die rolled will be 9. Justify your reasoning.

e. Using your lattice, determine the probability of rolling doubles.

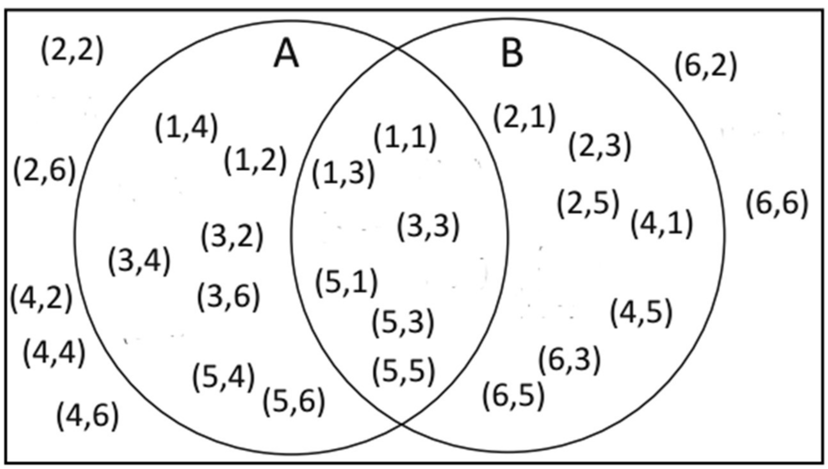
f. Using your lattice, determine the probability that the sum is a multiple of four.

4. The different outcomes that determine the probability of rolling odd can be visualized using a **Venn Diagram**, the beginning of which is seen below.

a. Explain the meaning of each pair represented in the Venn Diagram below.

A: ______________________________ B: ______________________________

b. Finish the Venn Diagram by placing the remaining 9 ordered pairs from the dice lattice in the appropriate place. Remaining ordered pairs: (1,5), (1,6), (2,4), (3,1), (3,5), (4,3), (5,2), (6,1), (6,4)



c. How many outcomes appear in circle A? (Hint: if ordered pairs appear in the overlap, they are still within circle A).

d. How many outcomes appear in circle B?

e. The portion of the circles that overlap (i.e. events that can occur at the same time) is called the **intersection**. The notation used for intersections is $\cap$, read as "A and B." How many outcomes are in $A \cap B$?

f. When you look at different parts of a Venn Diagram together, you are considering the **union** of the two outcomes. The notation for unions is $\cup$, read as "A or B." In the Venn Diagram you created, $A \cup B$ represents all the possible outcomes where an odd number shows. How many outcomes are in the union?

g. Record your answers to (c), (d), (e), and (f) in the table below.

c. Circle A	d. Circle B	e. $A \cap B$	f. $A \cup B$

h. Using the table above, how is your answer to e related (using mathematical operations) to b, c, and d?

i. Based on what you have seen, make a conjecture about the relationship of A, B, $A \cup B$ and $A \cap B$ using notation you just learned.

j. What outcomes have we not used yet and why?

Two events that have NO outcomes in common are called **mutually exclusive** (i.e. they cannot occur at the same time). Here are some examples:

- **Taking a M/C test by guessing:** The outcomes **getting the #1 correct** and **getting # 1 wrong** are Mutually Exclusive
- **Drawing a card from a standard deck:** The outcomes **Ace** and **Numbered Cards** are Mutually Exclusive
- **Rolling a die:** The outcomes **Even number** and **Odd number** are Mutually Exclusive.

Two events that have outcomes in common are sometimes referred to as **inclusive** (i.e. they can occur at the same time). Here are some examples:

- **Taking a M/C test by guessing:** The outcomes **getting the #1 correct** and **getting # 2 wrong** are Inclusive.
- **Drawing a card from a standard deck:** The outcomes **Ace** and **Red Card** are Inclusive.
- **Rolling a die:** The outcomes **Even number** and **Number greater than 3** are Inclusive.

For both mutually exclusive and inclusive events the addition rule can be applied:

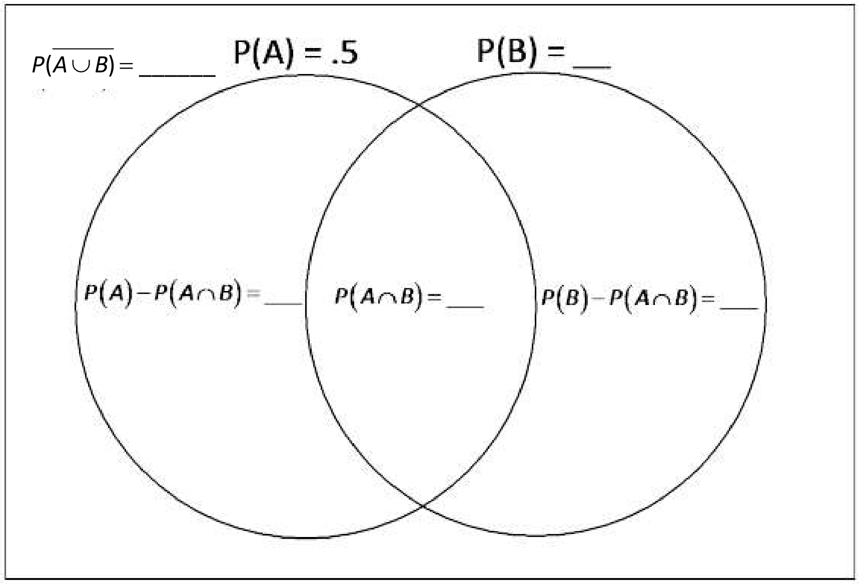
P(A or B) = P(A) + P(B) – P(A and B)

In a Venn Diagram the set of outcomes that are *not* included in some set is called the **complement** of that set. The notation used for the complement of set A is $\overline{A}$ (which is read "A bar") or A' (which is read "not A"). For example, in the Venn Diagram you completed above, the outcomes that are outside of $A \cup B$ are denoted $\overline{A \cup B}$.

k. Which outcomes appear in $A' - B$?

l. Which outcomes appear in $\left(\overline{A \cup B}\right)$?

5. Venn Diagrams can also be drawn using probabilities rather than outcomes. The Venn diagram below represents the probabilities associated with throwing two dice together. In other words, we will now look at the same situation as we did before, but with a focus on probabilities instead of outcomes.



a. Fill in the remaining probabilities in the Venn diagram.

b. Find $P(A \cup B)$ and explain how you can now use the probabilities in the Venn diagram rather than counting outcomes.

c. Use the probabilities in the Venn diagram to find $P(\overline{B})$.

d. What relationship do you notice between $P(B)$ and $P(\overline{B})$? Will this be true for any set and its complement?

__Example__: If P(C)= 0.7, what is the probability of P(C') ?

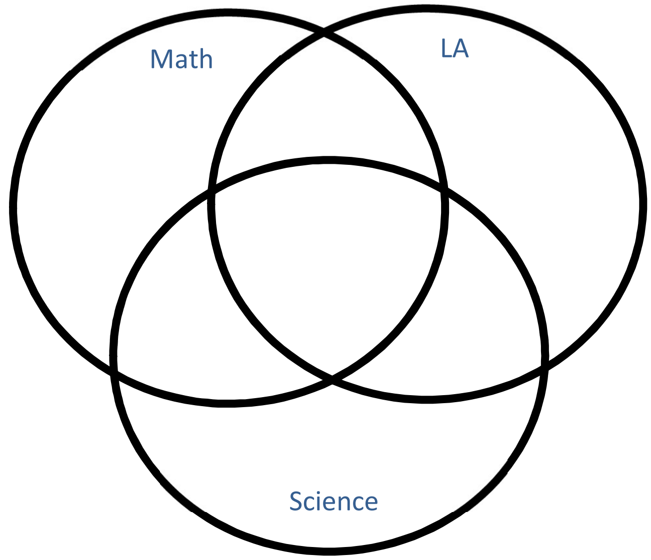
6. There are some sets that do not contain any element at all. For example, the set of months with 32 days. We call a set with no elements the **null** or **empty** set. It is represented by the symbol { } or Ø .
Some examples of null sets are:

- The set of squares with 5 sides.
- The set of integers which are both even and odd.

__**Example:**__ Let set A: {a,b,c,d,e} and B: {1,2,3,4}. List the outcomes that belong in the set $A \cap B$.

Guided Practice- Sets

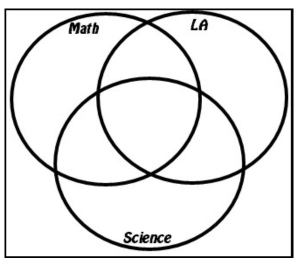
1. Make a VENN diagram of the following chart showing what classes each student was enrolled in this semester.



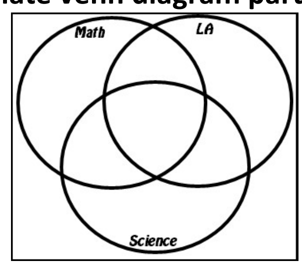
Name	Math	Language Arts	Science
Ashley	✓		✓
Betsy		✓	✓
Chris	✓	✓	✓
Devonte	✓	✓	
Eder	✓		✓
Frank			
George			✓
Heather		✓	
Isabella	✓		
Jessica		✓	
Krista	✓		

Using the venn diagram above, write each set and shade in the appropriate venn diagram part(s).

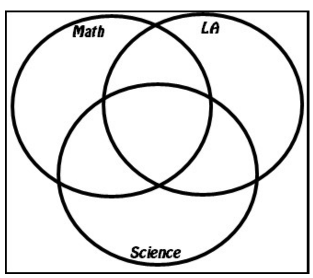
A. (LA):



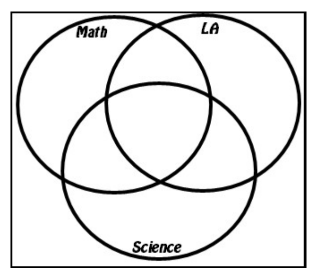
B. (Science):



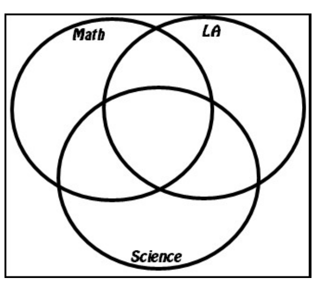
C. $(Math \cap Science)$:



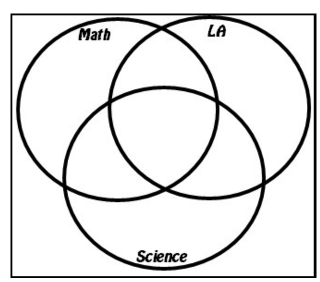
D. $(Math)'$:



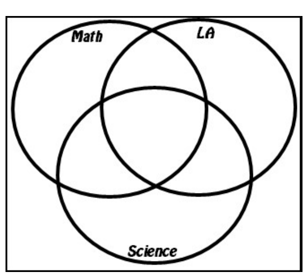
E. $(Math \cup LA)$:



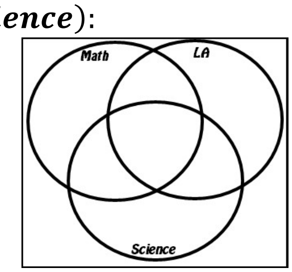
F. $(Math \cup LA)'$:



G. $(Math \cap LA')$:



H. $(Math \cap LA \cap Science)$:



2. Given A = { 1, 2, 3, 6, 7, 9} , B = { 2, 4, 6, 7, 8} , and Ω = { 1, 2, 3, 4, 5, 6, 7, 8, 9} answer the following.

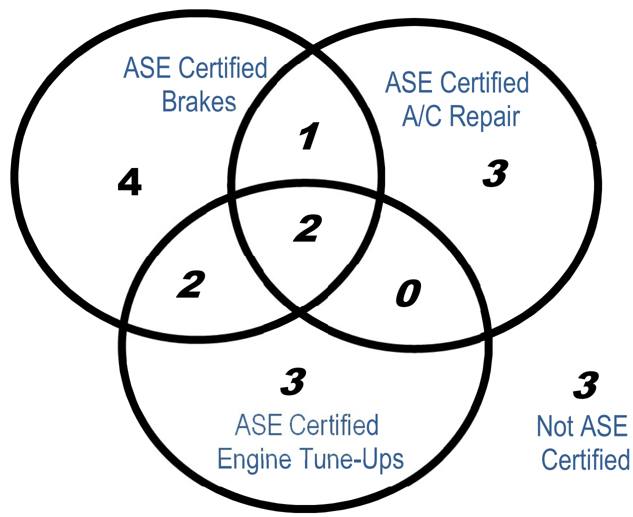
A. $(\boldsymbol{A} \cap \boldsymbol{B})$:

C. $(\boldsymbol{A} \cup \boldsymbol{B})$:

B. $(\boldsymbol{A})'$:

D. $(\boldsymbol{A} \cap \boldsymbol{B})'$:

3. A manager that owns 3 local area Car Maintenance Garages was researching certifications of mechanics that worked for her company. Consider the following Venn diagram.



a. How many mechanics worked for her company?

b. How many of the mechanics are certified by ASE to do work on Brakes?

c. How many of the mechanics are certified by ASE to do work on Brakes **and** Tune-Ups (Brakes $\cap$ Tune-Ups)?

d. How many of the mechanics are certified by ASE to do work on either A/C **or** Tune-Ups (A/C $\cup$ Tune-Ups)?

e. How many of the mechanics have their certification in Brakes **or** A/C but not in Tune-Ups???
$(Brakes\ \cup A/C)\ \cap\ (Tune\ Ups)'$

4. Elizabeth conducted a survey of students at her school about their favorite fast food restaurant (given three choices). Here are her results:

	Freshmen	**Sophomores**	**Juniors**	**Seniors**	**Total**
Zaxby's	31	26	20	22	
McDonald's	16	20	22	25	
Chick-Fil-A	45	36	40	30	
Total					

a. Calculate the totals of each row and column and fill in the table above. Also determine the total number of students who were surveyed in the bottom-right box of the table.

b. What is the probability that a randomly chosen student was a junior? P(J) =

c. What is the probability that a randomly chosen student was a senior? P(S) =

d. What is the probability that a randomly chosen student was a junior or a senior? P(J or S) =

e. What is the probability that a randomly chosen student preferred Zaxby's? P(Z) =

f. What is the probability that a randomly chosen student was a senior <u>and</u> preferred Zaxby's?
P(S and Z) =

g. What is the probability that a randomly chosen student was a senior <u>or</u> preferred Zaxby's?

P(S or Z) =

5. Consider the VENN diagrams at the right to help you answer the following.

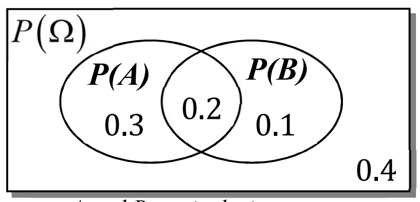
A. P(A) =

B. P(A and B) = $P(A \cap B)$=

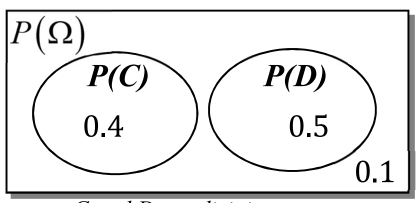
C. P(A or B) = $P(A \cup B)$=

D. P(A^c) = P(A') =

E. P(A and B^c) = $P(A \cap B')$ =



A and B are inclusive events.



C and D are disjoint events.

F. P(C) =

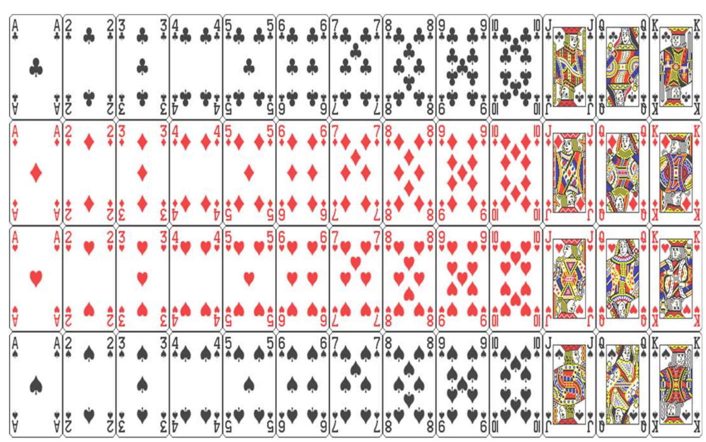
G. P(C and D) = $P(C \cap D)$=

H. P(C or D) = $P(C \cup D)$=

I. P(C^c) = P(C') =

J. P(C^c and D^c) = $P(C' \cap D')$=

6. There are 52 cards in a deck. There are 4 suits: Club, Spade, Diamonds, and Hearts. Clubs and Spades are black and Diamonds and Hearts are red. In each suit, there are 9 numbered cards (2-10), an Ace, and 3 face cards (King, Queen, and Jack).



a. What is the probability of randomly selecting a card from a standard 52 card deck and having the card be a **black card** or a **face card**?

Circle one of the following:	
Mutually Exclusive	Inclusive

b. What is the probability of randomly selecting a card from a standard 52 card deck and having the card be a **face card** or an **odd numbered card**?

Circle one of the following:	
Mutually Exclusive	Inclusive